MATHEMATICS 10

UNIT 3

EQUATIONS AND INEQUALITIES

Distance Learning

Alberta EDUCATION

Welcome

Distance Learning

You have chosen an alternate form of learning that allows you to work at your own pace. You will be responsible for your own schedule, for disciplining yourself to study the units thoroughly, and for completing your units regularly. We wish you much success and enjoyment in your studies.

Mathematics 10 Student Module Unit 3 Equations and Inequalities Alberta Distance Learning Centre ISBN No. 0-7741-0747-2

General Information

This information explains the basic layout of each booklet.

- **What You Already Know** and **Review** are to help you look back at what you have previously studied. The questions are to jog your memory and to prepare you for the learning that is going to happen in this unit.

- As you begin each **Topic**, spend a little time looking over the components. Doing this will give you a preview of what will be covered in the topic and will set your mind in the direction of learning.

- **Exploring the Topic** includes the objectives, concept development, and activities for each objective. Use your own papers to arrive at the answers in the activities.

- **Extra Help** reviews the topic. If you had any difficulty with **Exploring the Topic**, you may find this part helpful.

- **Extensions** gives you the opportunity to take the topic one step further.

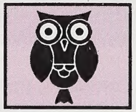
- To summarize what you have learned, and to find instructions on doing the unit assignment, turn to the **Unit Summary** at the end of the unit.

- The **Appendices** include the solutions to **Activities (Appendix A)** and any other charts, tables, etc. which may be referred to in the topics **(Appendix B, etc.)**.

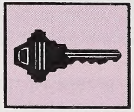
Visual Cues

Visual cues are pictures that are used to identify important areas of the material. They are found throughout the booklet.

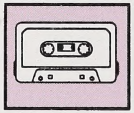
An explanation of what they mean is written beside each visual cue.



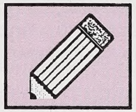
What You Already Know
- reviewing what you already know



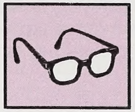
Key Idea
- flagging important ideas



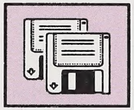
Audiotape
- learning by listening to an audiotape



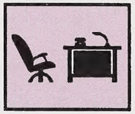
Review
- studying previous concepts



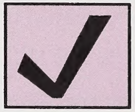
Another View
- exploring different perspectives



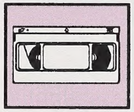
Computer Software
- learning by using computer software



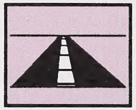
Introduction
- introducing the unit



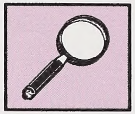
Solutions
- correcting the activities



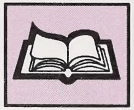
Videotape
- learning by viewing a videotape



What Lies Ahead
- previewing the unit

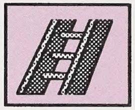


Extra Help
- providing additional study

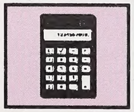


Print Pathway
- choosing a print alternative

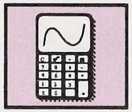
Exploring the Topic
- actively learning new concepts



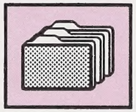
Extensions
- going on with the topic



Calculator
- using your calculator



Graphing Calculator
- using your graphing calculator



What You Have Learned
- summarizing what you have learned

Mathematics 10

Course Overview

Mathematics 10 contains 8 units. Beside each unit is a percentage that indicates what the unit is worth in relation to the rest of the course. The units and their percentages are listed below. You will be studying the unit that is shaded.

Unit 1 Number Systems	10%
Unit 2 Operations on Polynomials	14%
Unit 3 Equations and Inequalities	10%
Unit 4 Factoring Polynomials	13%
Unit 5 Coordinate Geometry	19%
Unit 6 Systems of Equations	10%
Unit 7 Trigonometry	11%
Unit 8 Statistics	13%
	100%

Unit Assessment

After completing the unit, you will be given a mark based totally on a unit assignment. This assignment will be found in the Assignment Booklet.

Unit Assignment - 100%

If you are working on a CML terminal, your teacher will determine what this assessment will be. It may be

Unit Assignment - 50%
Supervised Unit Test - 50%

Introduction to Equations and Inequalities

This unit covers topics dealing with equations and inequalities. Each topic contains explanations, examples, and activities to assist you in understanding equations and inequalities. If you find you are having difficulty with the explanations and the way the material is presented, there is a section called **Extra Help**. If you would like to extend your knowledge of the topic, there is a section called **Extensions**.

You can evaluate your understanding of each topic by working through the activities. Answers are found in the solutions in the **Appendix**. In several cases there is more than one way to do the question.

Unit 3 Equations and Inequalities

Contents at a Glance

Equations and Inequalities

The equation $s = vt$ tells you that displacement (s) is found by multiplying time (t) and average velocity (v). If a ball is dropped from a fifth-floor window, the equation $s = 4.9t^2$ tells how far this ball will drop in a given time (t). Equations or inequalities can be used to algebraically represent situations and relations that involve equalities or inequalities. If you can solve the equation or the inequality, you solve your problem.

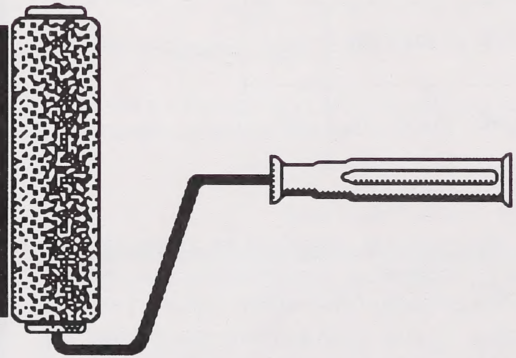
=	the sign of equality
< >	the signs of inequality

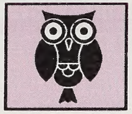
Unit 3
Equations and Inequalities

Topic 1 Solving and Verifying Linear Equations

Topic 2 Inequalities

Topic 3 Simple Quadratic and Radical Equations





What You Already Know

Recall the following:

The balance of an equation is maintained in the following situations.

- The same number is added to both sides.

  If $x = 5$, then $x + 3 = 5 + 3$.

Example 1

$x - 8 = 2$

Solution:

$$\begin{aligned} x - 8 &= 2 \\ x - 8 + 8 &= 2 + 8 \\ x &= 10 \end{aligned}$$

(Add 8 to both sides of the equation. Simplify by combining like terms.)

Verify: Substitute 10 back into the original equation. If the left side equals the right side, you know the answer is correct.

LS	RS
$x - 8$	2
$(10) - 8$	2
2	2

LS = RS

Since LS = RS, the solution is correct.

- The same number is subtracted from both sides.

  If $x = 5$, then $x - 3 = 5 - 3$.

Example 2

$x + 6 = 15$

Solution:

$$\begin{aligned} x + 6 &= 15 \\ x + 6 - 6 &= 15 - 6 \\ x &= 9 \end{aligned}$$

(Subtract 6 from both sides of the equation. Simplify by combining like terms.)

Verify: Substitute 9 for x in the original equation.

LS	RS
$x + 6$	15
$(9) + 6$	15
15	15

LS = RS

Since LS = RS, the solution is correct.

LS = Left Side
RS = Right Side

Recall: To verify means to **show** that your answer is correct.

- Each side is multiplied by the same nonzero number.

  If $x = 5$, then $3x = 5 \times 3$.

Example 3

$\frac{x}{2} = 6$

Solution:

$$\frac{x}{2} = 6$$

$$\frac{x}{2} \times 2 = 6 \times 2 \quad \text{(Multiply both sides by 2.)}$$

$$x = 12$$

Verify: Substitute 12 for x.

LS	RS
$\frac{x}{2}$	6
$\frac{(12)}{2}$	6
6	6
LS =	RS

- Each side is divided by the same nonzero number.

  If $x = 5$, then $\frac{x}{2} = \frac{5}{2}$.

Example 4

$4x = 28$

Solution:

$$4x = 28$$

$$\frac{4x}{4} = \frac{28}{4} \quad \text{(Divide both sides by 4.)}$$

$$x = 7$$

Verify: Substitute 7 for x.

LS	RS
$4x$	28
$4(7)$	28
28	28
LS =	RS

The inequality sign remains the same in the following situations.

- The same number is added to both sides of an inequality.

$$x > 3$$
$$x + 2 > 3 + 2$$

Example 5

$x - 5 < -4$

Solution:

$$x - 5 < -4$$
$$x - 5 + 5 < -4 + 5 \quad \text{(Add 5 to both sides and simplify.)}$$
$$x < 1$$

Verify: Since $x < 1$, substitute a number for x that is less than 1. Try -2.

LS	RS
$x - 5$	-4
$(-2) - 5$	-4
-7	-4

LS < RS

Since this statement is true, the solution is correct.

- The same number is subtracted from both sides of an inequality.

$$x > 3$$
$$x - 2 > 3 - 2$$

Example 6

$x + 2 > 9$

Solution:

$$x + 2 > 9$$
$$x + 2 - 2 > 9 - 2 \quad \text{(Subtract 2 from both sides.)}$$
$$x > 7$$

Verify: Since $x > 7$, substitute a number for x that is greater than 7. Try 8.

LS	RS
$x + 2$	9
$(8) + 2$	9
10	9

LS > RS

Since LS > RS, the solution is correct.

The symbol > means **greater than.**

The symbol < means **less than.**

The expression $x > 7,\ x \in I$ means that x is in the set of integers.

$8, 9, 10, \ldots$

The expression $x < -3,\ x \in I$ means that x is in the set of integers.

$-4, -5, -6, \ldots$ or $\ldots -6, -5, -4$

- Multiply both sides of an inequality by the same **positive** number.

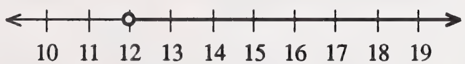
Example 7

$\frac{x}{3} > 4$

Solution:

$$(3)\left(\frac{x}{3}\right) > (3)(4)$$
$$x > 12$$

The graph of the situation is as follows:



Verify: Since $x > 12$, substitute a number greater than 12. Try 13.

LS	RS
$\frac{x}{3}$	4
$\frac{(13)}{3}$	4
$4\frac{1}{3}$	4
LS >	RS

Since LS > RS, the solution is correct.

- Divide both sides of an inequality by the same **positive** number.

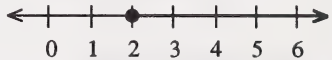
Example 8

$3x \geq 6$

Solution:

$$3x \geq 6$$
$$\frac{3x}{(3)} \geq \frac{6}{(3)}$$
$$x \geq 2$$

The graph of the situation is as follows:



Verify: Substitute 2 and a number greater than 2. Try $x = 3$.

$x = 2$

LS	RS
$3x$	6
$3(2)$	6
6	6
LS =	RS

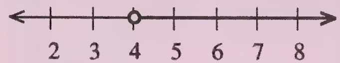
$x = 3$

LS	RS
$3x$	6
$3(3)$	6
9	6
LS >	RS

Since both conditions are met, the solution is correct.

Now that you have **looked** at material you studied previously, go to the **Review** to confirm your understanding of this material.

$x > 4$



The ○ means that 4 is not part of the solution. Only numbers greater than 4 will work.

$x \geq 4$

The ● means that 4 is included in the solution set.

The symbol ≥ means **greater than or equal to.**

The symbol ≤ means **less than or equal to.**

There are two conditions in $3x \geq 6$.

$3x \geq 6$

$3x = 6$ or $3x > 6$

Review

1. Solve $x + 3 = 8$.

2. Solve $x - 7 = 3$.

3. Solve $3x = 18$.

4. Solve $\frac{x}{4} = 2$.

5. Solve $3x - 3 = x + 5$.

6. Solve and graph $x + 1 > 3$.

7. Solve and graph $x - 3 \le 1$.

8. Solve and graph $\frac{x}{3} < 1$.

9. Solve and graph $5x \ge 15$.

Now go to the **Review** solutions in the **Appendix**.

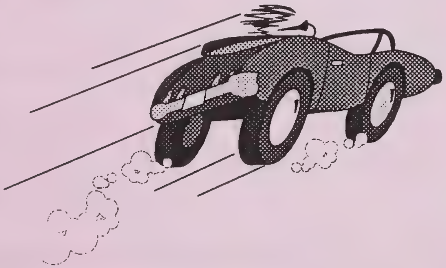
Topic 1 Solving and Verifying Linear Equations

Introduction

The average velocity of a uniformly accelerated car is half the sum of the initial and final velocities.

The equation $v_{av} = \frac{1}{2}v_1 + \frac{1}{2}v_2$ helps to describe the relationship between average velocity (v_{av}), initial velocity (v_1), and final velocity (v_2). If you know the initial and final velocities, you will be able to find the average velocity.

Equations are the tools you can use to solve such problems.



What Lies Ahead

Throughout this topic you will learn to

1. translate English sentences into algebra
2. solve and verify simple linear equations with integral coefficients
3. solve and verify simple linear equations with rational coefficients

Now that you know what to expect, turn the page to begin your study of solving and verifying linear equations.

Exploring Topic 1

Activity 1

Translate English sentences into algebra.

Mathematics is a universal language. Universal symbols are used to represent things such as variables, unknowns, and operations. For example, the symbol + is used to represent addition and x may be used to represent a certain number. It is important that you be able to translate English sentences into the language of algebra. If the facts of a problem can be translated into algebraic symbols, then you can solve the problem algebraically.

Now look at some operation symbols.

- The + symbol is equivalent to such phrases as the following:

  the sum of
  added to
  increased by
  more than

English Phrase or Sentence	Mathematical Phrase
the sum of x and five	$x+5$
Three is added to y.	$y+3$
four more than x	$x+4$
Seven is increased by a.	$7+a$

- The – sign is equivalent to such phrases as the following:

  the difference of
  less than
  decreased by
  subtracted from

English Phrase or Sentence	Mathematical Phrase
Two is subtracted from x.	$x-2$
the difference of x and y	$x-y$
Five is decreased by x.	$5-x$
nine less than a	$a-9$

- The $\times$ sign is equivalent to such phrases as the following:

the product of
times
multiplied by
of

English Phrase or Sentence	Mathematical Phrase
four times x	$4 \times x$ or $4x$
Five is multiplied by y.	$5 \times y$ or $5y$
three quarters of n	$\frac{3}{4} \times n$ or $\frac{3}{4}n$
the product of a and b	$a \times b$ or ab
x is doubled.	$2 \times x$ or $2x$

- The $\div$ sign is equivalent to such phrases as the following:

divided by
the quotient of
over

English Phrase or Sentence	Mathematical Phrase
a is divided by b.	$\frac{a}{b}$ or $a \div b$
the quotient of x and 3	$\frac{x}{3}$ or $x \div 3$
two over three	$\frac{2}{3}$ or $2 \div 3$

A mathematical phrase can be built step by step. In some cases, you may be asked to build a mathematical phrase, but the instructions may not be quite so clear and precise. You must sort out the instructions yourself and decide in which order the operations are to be performed on the unknown. For example, if you want to translate **eight more than the quotient of a certain number and five** into a mathematical phrase, the first step is to let x equal a certain number. Then the quotient of a certain number and 5 is $\frac{x}{5}$. Now 8 is added to the phrase to complete the translation. Thus, $\frac{x}{5}+8$ is the mathematical phrase you want.

Now try the following questions on your own.

1. Translate each of the following into a mathematical phrase.

   a. three more than a certain number

   b. seven less than a certain number

   c. eight divided by a certain number

   d. six times a certain number

   e. the difference of five and a certain number

   f. the value of t 42¢ stamps

   g. one less than twice a certain number

   h. three more than half a certain number

   i. the quotient of a certain number and three

   j. twice a certain number increased by five

For additional help with this section, you may use the Apple II diskette titled *Problem Solving in Algebra*[1], disk 1, lessons 1 to 7.

2. There are two quantities in each of the following situations. Choose a variable to represent one and express the other in terms of the first. Do at least two of the following problems.

   a. Jim is two times Sonja's age.

   b. Nadia earns $300 more per month than Kurtis.

   c. The length of a rectangle is 5 cm longer than the width.

   d. The number of boys in a class is four less than twice the number of girls.

For the solutions to **Activity 1**, turn to the **Appendix**, **Topic 1**.

[1] Problem Solving in Algebra is a title of *Britannica*.

Activity 2

Solve and verify simple linear equations with integral coefficients.

You have learned how to translate an English phrase into a mathematical phrase. Now move one step further. Suppose you have a practical problem. In order to solve the problem, you must be able to translate problem situations into mathematical phrases.

Always begin by letting some letter represent the unknown quantity of the problem. Then set up the equation and solve it by applying the skills you have acquired. In all cases in this unit, unless otherwise specified, the replacement set for the variable(s) is the set of real numbers. The following examples will show you the procedure.

Example 1

Amin is a dishwasher salesman. His monthly salary is $1000 plus $55 commission per dishwasher sold. His last pay cheque was $2925. How many dishwashers did he sell?

You may use any letter to represent a variable.

Hint: It is usually easier to let the variable represent the **lesser** of the two quantities being compared.

Solution:

Follow the four-step procedure.

Step 1: Understand the problem.
Let x be the number of dishwashers sold. Thus, the commission is equal to $\$55x$.

Step 2: Develop a plan.
The total salary can be expresses as $\$1000 + \$55x = \$2925$.

Step 3: Carry out the plan.
Solve the equation.

$$1000 + 55x - 1000 = 2925 - 1000 \quad \text{(Subtract 1000 from both sides.)}$$
$$55x = 1925$$
$$\frac{55x}{55} = \frac{1925}{55} \quad \text{(Divide both sides by 55.)}$$
$$x = 35$$

Step 4: Look back.
Verify that your answer fits the original conditions. Substitute $x = 35$ into the equation. State your answer in a sentence.

LS	RS
$1000 + 55(x)$	2925
$1000 + 55(35)$	2925
$1000 + 1925$	
2925	2925

LS = RS

Amin sold 35 dishwashers last month.

The problem-solving procedure is as follows:

Step 1: Understand the problem.
Step 2: Develop a plan.
Step 3: Carry out the plan.
Step 4: Look back.

LS = Left Side
RS = Right Side

One method of solving word problems is to use an equation. When solving equations, you will use the basic operations already studied, and sometimes you may have to apply properties such as the distributive property or the associative property.

The distributive property states that if there is a number outside the parentheses, multiply everything inside the parentheses by that number to remove the parentheses. This property is used in the following examples.

Example 2

Solve $3(x-2)+x=5(x-1)+3$.

Solution:

$$\begin{aligned}
3(x-2)+x&=5(x-1)+3 &&\text{(Use the distributive property.)}\\
3x-6+x&=5x-5+3\\
4x-6&=5x-2 &&\text{(Simplify.)}\\
4x-6-5x&=5x-2-5x &&\text{(Subtract } 5x \text{ from both sides.)}\\
-x-6&=-2\\
-x-6+6&=-2+6 &&\text{(Add 6 to both sides.)}\\
-x&=4\\
\frac{-x}{-1}&=\frac{4}{-1} &&\text{(Divide both sides by } (-1)\text{.)}\\
x&=-4
\end{aligned}$$

Verify:

LS	RS
$3(x-2)+x$	$5(x-1)+3$
$3(-4-2)+(-4)$	$5(-4-1)+3$
$3(-6)-4$	$5(-5)+3$
$-18-4$	$-25+3$
-22	-22

LS = RS

Note in Example 2 how the distributive property is used to remove the parentheses.

$$\begin{aligned}
3(x-2)&=3(x)-3(2)\\
&=3x-6
\end{aligned}$$

$$\begin{aligned}
5(x-1)&=5(x)-5(1)\\
&=5x-5
\end{aligned}$$

A more involved example can include multiplying factors.

Example 3

Solve $x(x-3)+2=(x+2)(x-3)$.

Solution:

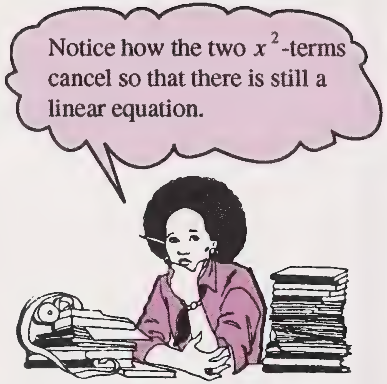
$x(x+3)+2=(x+2)(x-3)$	(Remove brackets by multiplying factors.)
$x^2-3x+2=x^2+2x-3x-6$	
$x^2-3x+2=x^2-x-6$	(Simplify.)
$x^2-x^2-3x+2=x^2-x^2-x-6$	(Subtract x^2 from both sides.)
$-3x+2=-x-6$	
$-3x+x+2-2=-x+x-6-2$	(Add x and subtract 2 from both sides.)
$-2x=-8$	
$\frac{-2x}{-2}=\frac{-8}{-2}$	(Divide both sides by (-2).)
$x=4$	

Verify:

LS	RS
$x(x-3)+2$	$(x+2)(x-3)$
$4(4-3)+2$	$(4+2)(4-3)$
$4(1)+2$	$6(1)$
$4+2$	6
6	6

LS = RS

Therefore, $x=4$ is the solution to the equation.



The next example involves word problems with linear equations with parentheses.

Example 4

Peter is a part-time worker. For the last three weeks, he earned \$1373. Peter earned \$25 more the first week than the second week. He earned \$2 less the third week than the second week. How much did Peter earn each week?

Solution:

Step 1: Understand the problem.
Let x be the amount earned in the second week.
Then $x+25$ is the amount earned in the first week, and
$x-2$ is the amount earned in the third week.

The x is chosen to represent the second week's earnings because the earnings for the first week and the third week are related to the earnings of the second week.

Step 2: Develop a plan.

$$(x+25)+x+(x-2)=\$1373$$

Step 3: Carry out the plan.

$$\begin{aligned} x+25+x+x-2 &= 1373 \\ 3x+23 &= 1373 && \text{(Simplify.)} \\ 3x+23-23 &= 1373-23 && \text{(Subtract 23 from both sides.)} \\ 3x &= 1350 \\ \frac{3x}{3} &= \frac{1350}{3} && \text{(Divide both sides by 3.)} \\ x &= 450 \end{aligned}$$

If $x=450$, then $x+25=475$ and $x-2=448$.

Step 4: Look back.

Substitute $x = 450$ into the original equation.

LS	RS
$(x+25)+x+(x-2)$	1373
$[(450)+25]+(450)+[(450)-2]$	1373
$475+450+448$	1373
1373	1373

LS = RS

Therefore, Peter earned $475, $450, and $448 in the three weeks.

There are many different types of problems. It is impossible to show all of them. The following are two more examples.

One important type deals with consecutive integers. Consecutive means one after the other.

Examples of consecutive integers are as follows:

- 2, 3, 4
- $-6, -5, -4$
- 124, 125, 126, 127

Note that each number is one more than the previous number.

If n represents one of the numbers, the next larger one may be written as $n+1$, the next as $n+2$, and so on.

Examples of consecutive even integers are as follows:

- 0, 2, 4, 6
- $-10, -8, -6$

Each number is two more than the previous number.

Therefore, if x is the first even integer, then $x+2$ is the next larger one, $x+4$ is the next one, and so one.

What about consecutive odd integers? Examine the following examples.

- 1, 3, 5, 7
- $-13, -11, -9, -7$
- 103, 105, 107

Each integer is _______ more than the previous integer.

If the first integer is x, the next two are _______ and _______ .

Since consecutive odd integers also have a difference of 2, each is **two** more than the one previous. If the first integer is x, the next two are $\boldsymbol{x+2}$ and $\boldsymbol{x+4}$.

Now work through the following example.

Example 5

Find two consecutive even integers such that the difference of four times the smaller and two times the larger is 16.

Solution:

Step 1: Understand the problem.
Let x be the smaller integer.
Then $x+2$ is the larger integer.

$4(x)=4$ times the smaller
$2(x+2)=2$ times the larger

Step 2: Develop a plan.

$$4(x)-2(x+2)=16$$

Step 3: Carry out the plan.

$$\begin{aligned} 4x-2x-4&=16 && \text{(Use the distributive property.)} \\ 2x-4&=16 && \text{(Simplify.)} \\ 2x-4+4&=16+4 && \text{(Add 4 to both sides.)} \\ 2x&=20 \\ \frac{2x}{2}&=\frac{20}{2} && \text{(Divide both sides by 2.)} \\ x&=10 \end{aligned}$$

$$\begin{aligned} -2(x+2)&=-2(x)-2(2) \\ &=-2x-4 \end{aligned}$$

Step 4: Look back.

LS	RS
$4(x)-2(x+2)$	16
$4(10)-2[(10)+2]$	16
$40-2(12)$	16
$40-24$	16
16	16

LS = RS

If the smaller number is x and the larger number is $x+2$, the two numbers are 10 and 12.

Another common type of problem is the age problem.

Three ages may be considered:

- present age which is represented by a variable such as x
- past age which is shown by subtraction
- future age which is shown by addition

Example 6

If you are n years old now, how old were you six years ago?

Solution:

Your age now is n.
Your age six years ago is $n-6$.

Example 7

If you are n years old now, how old will you be in five years?

Solution:

Your age now is n.
Your age five years from now is $n+5$

The representation of present, past, and future ages may vary depending on what you choose to let the variable represent.

Study the following chart.

Present Age	Age Five Years Ago	Age Three Years From Now
x	present age -5 $=x-5$	present age $+3$ $=x+3$
past age $+5$ $=x+5$	x	present age $+3$ $=(x+5)+3$ $=x+8$
future age -3 $=x-3$	present age -5 $=(x-3)-5$ $=x-8$	x

Example 8

Maxine is ten years younger than Mark. Two years from now, Mark will be twice as old as Maxine. How old is Mark now?

Solution:

Step 1: Understand the problem.
Let x be Maxine's age now.
Then $x+10$ is Mark's age.
Two years from now their ages are as follows:
Maxine's age $= x+2$

$$\begin{aligned}\text{Mark's age} &= (x+10)+2\\ &= x+12\end{aligned}$$

You may use a chart to organize your information.

	Present Age	Future Age $(+2)$
Maxine	x	$x+2$
Mark	$x+10$	$(x+10)+2=x+12$

Step 2: Develop a plan.
$(x+2)=2(x+2)$ (Two years from now, Mark's age is $2\times$Maxine's age.)

You could solve this problem another way by letting Mark's age be x. Then you would get the following elements:

Mark's age is x; thus, Maxine's age is $x-10$.

In two years Maxine's age will be $x-10+2=x-8$. Mark's age will be $x+2$. In two years Mark will be two times Maxine's age.

$$\begin{aligned}x+2&=2(x-8)\\ x+2&=2x-16\\ x+2+16&=2x\\ 18&=2x-x\\ 18&=x\end{aligned}$$

Thus, Mark is 18.

Step 3: Carry out the plan.

$$x+12=2x+4$$
$$x+12-x=2x+4-x \quad \text{(Subtract } x \text{ from both sides.)}$$
$$12=x+4$$
$$12-4=x+4-4 \quad \text{(Subtract 4 from both sides.)}$$
$$8=x$$

Then $x=8$ and $x+10=18$.

Step 4: Look back.

LS	RS
$x+12$	$2(x+2)$
$(8)+12$	$2[8+2]$
20	$2(10)$
20	20
LS =	RS

Since Mark's age is $x+10$, he must be eighteen years old.

Now you should be ready to try some questions.

1. Solve and verify. Do a, b, and c or d.

   a. $3x-2(x+4)=5-3(x-1)$

   b. $1+3(x-5)=x-2(x+3)$

   c. $x(x+1)+3=(x-1)(x+5)$

   d. $(9x+1)(x-2)=(3x-1)^2$

Do any four of the following five questions.

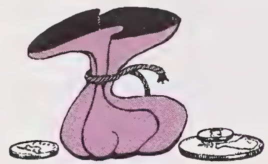
2. The larger of two numbers is eight more than two times the smaller number. If their difference is fifteen, find each number.

3. A mother is eight years older than three times her son's age. Four years ago, she was eleven times as old as her son. How old is the mother?

4. In ΔABC, the measure of $\angle A$ is 5° more than the measure of $\angle B$. The measure of $\angle C$ is 35° less than the measure of $\angle B$. What is the measure of each angle? (Remember that the sum of the measures of the three angles of any triangle is always 180°. A diagram may also be helpful in organizing the information.)

5. The length of a rectangle is 5 cm longer than the width. If the perimeter of the rectangle is 90 cm, find the dimensions of the rectangle. (Recall that $P=2l+2w$.)

6. In a bag of coins there are five more dimes than nickels and two fewer quarters than nickels. If the coins are worth a total of \$2.00, find the number of each kind of coin.



For solutions to **Activity 2**, turn to the **Appendix, Topic 1**.

Activity 3

Solve and verify simple linear equations with rational coefficients.

So far, all equations you have encountered have involved integral coefficients. However, many equations and problems involve fractional (rational) coefficients.

When dealing with equations that have rational coefficients you will need to **multiply every term by a common denominator** so that you have an equation without fractions. Study the following example.

Example 9

Solve and verify $\frac{x}{5} = 4$.

Solution:

Multiply every term by 5.

$$\frac{x}{5}(5) = 4(5)$$

$$x = 20$$

See if the value you obtained for x makes the equation true. Substitute 20 for x.

LS	RS
$\frac{x}{5}$	4
$\frac{20}{5}$	4
4	4

LS = RS

A **coefficient** is the numerical value that is multiplying the variable.

Think of a rational number as being in fractional form.

In the next example, variables and fractions will appear on both sides.

Example 10

Solve and verify $\frac{3(x-1)}{2}+\frac{x}{3}=\frac{x}{5}$.

Solution:

Multiply every term by the lowest common denominator (L.C.M.) of 2, 3, and 5, which is 30.

$$\frac{3(x-1)}{2}+\frac{x}{3}=\frac{x}{5}$$

$$\overset{15}{\cancel{(30)}}\left(\frac{3(x-1)}{\underset{1}{\cancel{2}}}\right)+\overset{10}{\cancel{(30)}}\left(\frac{x}{\underset{1}{\cancel{3}}}\right)=\overset{6}{\cancel{30}}\left(\frac{x}{\underset{1}{\cancel{5}}}\right)$$

$$15[3(x-1)]+10(x)=6(x) \quad \text{(Use the distributive property.)}$$

$$15(3x-3)+10(x)=6(x) \quad \text{(Use the distributive property.)}$$

$$45x-45+10x=6x \quad \text{(Simplify by collecting like terms.)}$$

$$55x-45=6x$$

$$55x-45+45=6x+45 \quad \text{(Add 45 to both sides.)}$$

$$55x=6x+45$$

$$55x-6x=6x+45-6x \quad \text{(Subtract } 6x \text{ from both sides.)}$$

$$49x=45 \quad \text{(Divide both sides by 49.)}$$

$$x=\frac{45}{49}$$

L.C.M. = lowest common multiple

L.C.M. of 2, 3, and $5=2\times3\times5$
$=30$

Verify:

LS	RS
$\dfrac{3(x-1)}{2}+\dfrac{x}{3}$	$\dfrac{x}{5}$
$\dfrac{3\left(\frac{45}{49}-1\right)}{2}+\dfrac{\frac{45}{49}}{3}$	$\dfrac{\frac{45}{49}}{5}$
$\dfrac{3\left(\frac{45-49}{49}\right)}{2}+\dfrac{45}{49}\div 3$	$\dfrac{45}{49}\div 5$
$\dfrac{3\left(\frac{-4}{49}\right)}{2}+\dfrac{45}{49}\times\dfrac{1}{3}$	$\dfrac{45}{49}\times\dfrac{1}{5}$
$\dfrac{-12}{49}\times\dfrac{1}{2}+\dfrac{15}{49}$	$\dfrac{9}{49}$
$\dfrac{-6+15}{49}$	$\dfrac{9}{49}$
$\dfrac{9}{49}$	$\dfrac{9}{49}$

LS = RS

If you had trouble working with the L.C.M., go to the **Extra Help** section.

Now look at a word problem that uses rational coefficients.

Example 11

Sarah, Teresa, and Masami shared a box of chocolate bars. Sarah took $\frac{1}{3}$ of the box, Teresa took $\frac{1}{4}$ of the box, and Masami took $\frac{1}{6}$ of the box. After they took their shares, there were six chocolate bars left. How many chocolate bars were there in the first place?

Solution:

Understand the problem.
Let x be the total number of chocolate bars in the box.

Sarah took $\frac{1}{3}x$ chocolate bars.

Teresa took $\frac{1}{4}x$ chocolate bars.

Mesami took $\frac{1}{6}x$ chocolate bars.

Develop a plan.

$$\frac{1}{3}x+\frac{1}{4}x+\frac{1}{6}x+6=x$$

Carry out the plan.
The lowest common denominator of 3, 4, and 6 is 12.
Multiply every term by 12.

$$\overset{4}{\cancel{(12)}}\left(\frac{1}{\underset{1}{\cancel{3}}}x\right)+\overset{3}{\cancel{(12)}}\left(\frac{1}{\underset{1}{\cancel{4}}}x\right)+\overset{2}{\cancel{(12)}}\left(\frac{1}{\underset{1}{\cancel{6}}}x\right)+(12)(6)=(12)x$$

$$4x+3x+2x+72=12x$$

$$9x+72=12x$$

$$72=3x$$

$$\frac{72}{3}=x$$

$$24=x$$

Look back.
Substitute $x=24$ into the original equation.

LS	RS
$\frac{x}{3}+\frac{x}{4}+\frac{x}{6}+6$	x
$\frac{24}{3}+\frac{24}{4}+\frac{24}{6}+6$	24
$8+6+4+6$	24
24	24

LS = RS

Therefore, there were 24 chocolate bars.

You may find the Apple II diskette titled *Solving Fractional Equations*[1] useful when doing this section.

[1] Solving Fractional Equations is a title of *Mindscape Inc.*

If you feel confident that you understand the examples, do the following questions.

1. Solve and verify the following. Do either a and c or b and d.

   a. $\frac{5}{7}m=8$

   b. $\frac{x}{3}+5=\frac{x}{2}-1$

   c. $\frac{(x-2)}{3}-\frac{(x+1)}{5}=4$

   d. $\frac{2x-3}{4}=\frac{x+5}{3}$

Do either 2 and 4, or 3 and 5.

2. Brenda has 5 L of 70% sulphuric acid solution. In order to make a concentration of 20% sulphuric acid solution, Brenda must add the acid solution to water. How much water will she need?

3. One number exceeds another by 6. The sum of $\frac{1}{3}$ of the smaller number and $\frac{1}{2}$ of the larger number is 33. Find the two numbers.

4. The difference of two numbers is 4. The sum of $\frac{1}{2}$ of the larger number and two times the smaller number is 12. Find the two numbers.

5. Helmut has 1 L of 15% acetic acid (vinegar) solution. In order to make a concentration of 10% acetic acid solution, Helmut must add the vinegar solution to water. How much water will he need?

For solutions to **Activity 3**, turn to the **Appendix**, **Topic 1**.

If you require help, do the Extra Help section.

If you want more challenging explorations, do the Extensions section.

} You may decide to do both.

Extra Help

Lowest Common Multiple (L.C.M.)

There are different methods you can use to determine the lowest common multiple of a set of numbers. The method shown here may be new to you.

Example 12

Find the L.C.M. of 2 and 6.

Solution:

This method is similar to short division. Use short division to divide the two numbers by their common factor. The L.C.M. is the product of the divisor and the quotients.

2 is a common factor of 2 and 6. $\rightarrow$

$$\begin{array}{r|ll} 2 & 2 & 6 \\ \hline & 1 & 3 \end{array} \quad \begin{array}{l} \leftarrow \text{Write down the numbers.} \\ \leftarrow \text{quotients} \end{array}$$

$$\begin{aligned} \text{L.C.M.} &= 2\times1\times3 \\ &= 6 \end{aligned}$$

Example 13

Find the L.C.M. of 4, 8, and 12.

Solution:

4 is a common factor of 4, 8, and 12. $\rightarrow$

$$\begin{array}{r|lll} 4 & 4 & 8 & 12 \\ \hline & 1 & 2 & 3 \end{array} \quad \begin{array}{l} \leftarrow \text{Write down the numbers.} \\ \leftarrow \text{quotients without common factors. (relatively prime)} \end{array}$$

$$\begin{aligned} \text{L.C.M.} &= 4\times1\times2\times3 \\ &= 24 \end{aligned}$$

Example 14

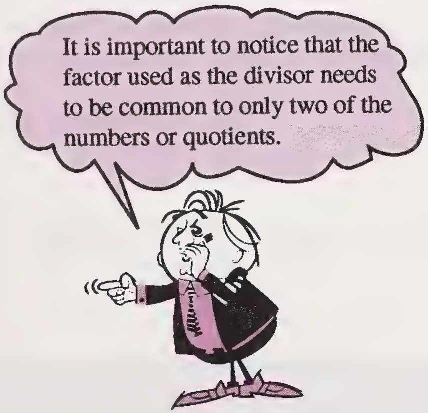
Find the L.C.M. of 4, 8, and 24.

Solution:

common factor	→ 4	4	8	24	← Write down the numbers.
If you can find another common factor of any two of the quotients, divide them by that common factor.	→ 2	1	2	6	← quotients.
		1	1	3	← quotients without common factors. (except 1)
		↑			

2 is not a common factor of 1. Do not divide 1 by 2. Leave it here.

$$\text{L.C.M.} = 4 \times 2 \times 1 \times 1 \times 3$$
$$= 24$$



Example 15

Find the L.C.M. of 15, 20, and 30.

Solution:

common factor of 15, 20, and 30	→ 5	15	20	30	← Write down the numbers.
common factor of 3 and 6	→ 3	3	4	6	← Do not divide 4 by 3. Leave the number 4 alone.
common factor of 4 and 2	→ 2	1	4	2	
		1	2	1	← quotients which are relatively prime
		↑			

Leave the number 1 here.

$$\text{L.C.M.} = 5 \times 3 \times 2 \times 1 \times 2 \times 1$$
$$= 60$$

It is your turn to try. Do any three of the following four questions.

1. Find the L.C.M. of 10 and 5.
2. Find the L.C.M. of 8, 10, and 15.
3. Find the L.C.M. of 8, 12, and 18.
4. Find the L.C.M. of 4, 10, 15, and 18.

For solutions to **Extra Help**, turn to the **Appendix**, **Topic 1**.

Extensions

What you have learned in this topic involves linear equations with one unknown. If a linear equation has two unknowns, such as $2x+3y-7=0$, you need another equation to form a system of equations. Then you can solve the system of equations for the unknowns. You are going to learn more about system of equations in **Unit 6**.

In this topic you have worked with abstract number problems, coin problems, business problems, age problems, measurement problems, and mixture problems. In this section you will find some word problems that are different. Try to solve all types of problems using the four-step procedure to develop your problem-solving skill.

Example 16

Orest makes \$95 for every day he works and forfeits \$40 for every day he does not work. Last month, at the end of 30 days, his pay was \$2175. How many days did he work?

Solution:

Step 1: Let x be the days he worked.
Then, $30-x$ are the days he did not work.
Amount earned = $\$95x$
Amount forfeited = $\$40(30-x)$

Step 2: Salary = amount earned – amount forfeited

$$2175 = 95x - 40(30-x)$$

Step 3:
$$\begin{aligned} 95x-1200+40x &= 2175 \\ 135x-1200 &= 2175 \\ 135x &= 3375 \\ x &= \frac{3375}{135} \\ &= 25 \end{aligned}$$

Verify:

LS	RS
$95(25)-1200+40(25)$	2175
$2375-1200+1000$	2175
2175	2175

LS = RS

Therefore, Orest worked 25 days.

The next example is a digit problem.

Example 17

Find a two-digit number such that the first digit exceeds the second digit by three. When the digits are reversed, the new number is $\frac{4}{7}$ times the original number.

Solution:

Step 1: Let x be the unit digit.
Then, $x+3$ is the tens digit.
Original number $= 10(x+3)+x$ (Note: A tens digit has a value of the digit times 10.)
After reversing the two digits,
New unit digit $= (x+3)$
New tens digit $= x$
New number $= 10x+(x+3)$

Step 2: $10x+(x+3) = \frac{4}{7}[10(x+3)+x]$

Step 3:
$$\begin{aligned} 10x+x+3 &= \frac{4}{7}(10x+30+x) \\ 11x+3 &= \frac{4}{7}(11x+30) \\ 77x+21 &= 44x+120 \\ 33x &= 99 \\ x &= 3 \end{aligned}$$

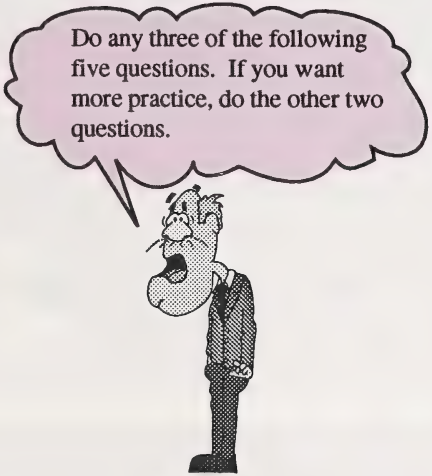
$$\begin{aligned} \therefore x+3 &= 3+3 \\ &= 6 \end{aligned}$$

Step 4: Substitute 3 into Step 2.

LS	RS
$10(3)+[(3)+3]$	$\frac{4}{7}(10[(3)+3]+3)$
$30+6$	$\frac{4}{7}(60+3)$
36	$\frac{4}{7}(63)$
36	4×9
36	36

LS = RS

The unit digit is 3 and the tens digit is 6.
Therefore, the number is 63.



1. If (-4) is a root of the equation $3(x+k)-2(x-1)=k+2$, find k.

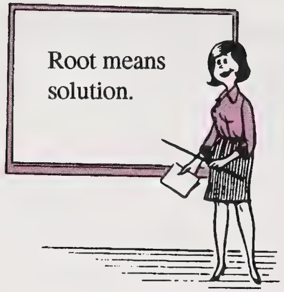
2. Solve and verify $\frac{5}{x}-\frac{1}{x}=2$.

3. Mr. Taylor is three times as old as his daughter. Ten years ago, Mr. Taylor was four times as old as his daughter. How old is Mr. Taylor?

4. Two motorcycles can complete a loop on a circular 2 km track in 15 s and 20 s, respectively. If they start at the same time and the same place, but in opposite directions, how many seconds will have passed when they meet each other? Round your answer to the nearest hundredth.

5. Find a two-digit number if the sum of the two digits is nine and the units digit is five less than the tens digit.

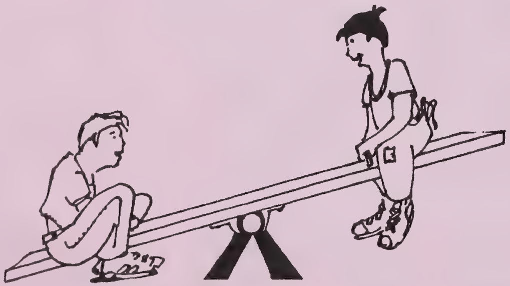
For solutions to **Extensions**, turn to the **Appendix**, **Topic 1**.



Topic 2 Inequalities

Introduction

Changing a mathematics symbol relating two expressions changes the meaning. If there is an equal sign between two expressions such as A and B, they form an equation $A = B$. If A and B are not equal (A is greater than or less than B), then they form an inequality either $A > B$ or $A < B$. The symbols $>$, $\geq$, $<$, $\leq$, and $=$ are all important symbols in the language of mathematics. If these symbols are not used correctly, they can result in incorrect answers. In this topic you will learn how to apply the skills you have learned with equations and see how they can be applied and adjusted, where necessary, in solving inequalities.



What Lies Ahead

Throughout this topic you will learn to

1. solve, graph, and verify any linear inequalities which have rational coefficients, and apply the reverse-the-sign rule as needed
2. solve word problems with linear inequalities

Now that you know what to expect, turn the page to begin your study of inequalities.

Exploring Topic 2

Activity 1

Solve, graph, and verify any linear inequalities which have rational coefficients, and apply the reverse-the-sign rule as needed.

If the masses of two boxes *A* and *B* are equal and box *A* has a mass of 5 kg, then box *B* must have a mass of 5 kg. If the masses of these two boxes are not equal and box *A* has a mass of 5 kg, then the mass of box *B* can be 2 kg, 4 kg, 6 kg, 7 kg, and so on. There is an infinite number of possibilities. You will not be able to write down all the solutions. You have to use the greater than (>) sign, the less than (<) sign, or a graph to represent the whole set of answers. An inequality is formed when two expressions are connected by one of the following inequality symbols:

$>$ greater than
$<$ less than
$\geq$ greater than or equal to
$\leq$ less than or equal to

For example, $2x > 7$ and $x - 3 \leq 3x + 2$ are examples of linear inequalities. In order to determine the value of *x*, you have to solve the inequality. How is this done? Do the rules for working with equations apply to inequalities?

Remember that when you add, subtract, multiply, or divide both sides of an equation by the same positive or negative number, the two sides of the equation remain equal. Would these rules apply to inequalities as well?

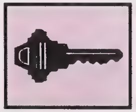
Consider the two numbers 9 and 15. They can be related in the inequality $15 > 9$. The following table explores the results when various operations are applied to $15 > 9$.

Operation	LS	RS	Is LS > RS?
Add three to both sides.	18	12	yes
Add negative three to both sides.	12	6	yes
Subtract three from both sides.	12	6	yes
Subtract negative three from both sides.	18	12	yes
Multiply both sides by 3.	45	27	yes
Multiply both sides by negative three.	-45	-27	no
Divide both sides by three.	5	3	yes
Divide both sides by negative three.	-5	-3	no

LS = Left Side
RS = Right Side

This table suggests that most of the rules for solving equations apply to inequalities, but two do not. After you multiply both sides by (-3), the left side is -45 and right side is -27. Since $-45 < -27$, the left side is no longer greater than the right side. You have to reverse the inequality sign in order to keep the inequality true. The same thing happens if you divide both sides by a negative number. The key idea is shown here.

The number -45 is to the left of -27. Therefore, $-45 < -27$.



Reverse-the-Sign Rule
When multiplying or dividing both sides of an inequality by a negative number, you must reverse the inequality sign.

Are you ready to see how inequalities are solved?

Example 1

Solve $2x+3>7+x$.

Solution:

$$2x+3-3>7-3+x \quad \text{(Subtract 3 from both sides.)}$$
$$2x>4+x$$
$$2x-x>4+x-x \quad \text{(Subtract } x \text{ from both sides.)}$$
$$x>4$$

The solution $x>4$ means that any number greater than 4 would satisfy the inequality. To verify this solution, you can choose any such number and test it. Try the number 5.

LS		RS
$2x+3$		$7+x$
$2(5)+3$		$7+(5)$
13		12
LS	$>$	RS

Since $13>12$, you have shown that the left side is greater than the right side. When $x=5$, the inequality remains the same as the original inequality.

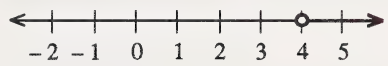
Now try the number 3, which is less than 4.

LS		RS
$2x+3$		$7+x$
$2(3)+3$		$7+(3)$
9		10
LS	$\not>$	RS

Since 9 is not greater than 10, the inequality is not true for $x=3$.

It appears from these tests that the inequality is true for $x>4$.

There is another way to illustrate this solution. Graphs are helpful. Thus, this solution can be illustrated using a number line.



This number-line illustration shows the graph of the inequality $x>4$.

Now look at an example that has a negative value with the variable.

A hollow circle is used because the point 4 is not included.

Example 2

Solve, verify, and graph $3(x-1)-7x \le 9$.

Solution:

$3(x-1)-7x \le 9$	(Use the distributive property.)
$3x-3-7x \le 9$	
$-4x-3 \le 9$	(Simplify.)
$-4x-3+3 \le 9+3$	(Add 3 to both sides.)
$-4x \le 12$	(Simplify.)
$\frac{-4x}{-4} \ge \frac{12}{-4}$	(Divide both sides by -4 and reverse the inequality sign.)
$x \ge -3$	

Verify:

Let $x=0$.

LS	RS
$3(x-1)-7x$	9
$3[(0)-1]-7(0)$	9
$3(-1)-0$	9
-3	9

LS $<$ RS

Since $-3 \le 9$, the inequality is true when $x=0$.

Now let $x=-4$, which is less than -3.

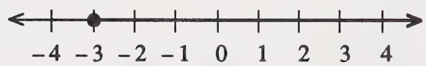
LS	RS
$3(x-1)-7x$	9
$3(-4-1)-7(-4)$	9
$-15+28$	9
13	9

LS $\not\le$ RS

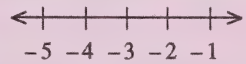
Since 13 is not less than 9, the inequality is not true when $x=-4$.

These tests show that the inequality is true for $x \ge -3$.

Graph:



The next example has rational coefficients. Multiply every term by the lowest common denominator and you will be able to eliminate all the fractions.



The number -4 is to the left of -3. Therefore, $-4<-3$.

Reverse the inequality sign since you are dividing by a negative value.

This is a solid dot because -3 is included.

Example 3

Solve and graph $2-\frac{5x}{7}>\frac{1}{2}$.

Solution:

$$(14)2-(14)\frac{5x}{7}>\frac{1}{2}(14)$$ (Multiply every term by 14.)

$$28-10x>7$$

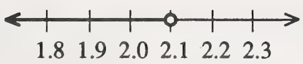
$$28-10x-28>7-28$$ (Subtract 28 from both sides.)

$$-10x>-21$$

$$\frac{-10x}{-10}<\frac{-21}{-10}$$ (Divide both sides by -10 and reverse the inequality sign.)

$$x<+\frac{21}{10}$$

$$x<2.1$$



You may want to review the previous examples as you do the following questions.

Do either part a or b in questions 1, 2 and 3. If you need more practice, do the other part as well. Do both parts in question 4.

1. Solve, graph, and verify.

   a. $5x-7\leq 3x+11$

   b. $3x+2\geq 14-x$

2. Solve, graph, and verify.

   a. $3(4-x)>x+8$

   b. $2(2-x)\leq 4(x-14)$

3. Solve and graph.

   a. $\frac{x}{2}-\frac{x}{3}\geq -\frac{5}{6}$

   b. $\frac{1}{4}<\frac{x}{2}-\frac{x}{3}$

4. Solve and graph.

   a. $\frac{(3-4x)}{2}\leq\frac{2(x-3)}{5}$

   b. $(x+2)(3x-1)\leq 3x(x+2)-4$

For solutions to **Activity 1**, turn to the **Appendix**, **Topic 2**.

Activity 2

Solve word problems with linear inequalities.

Word problems with linear inequalities are similar to those with linear equations. Here is an example. Read it carefully.

Example 4

Jack is a salesman. His monthly salary is \$800 plus 10% commission on his total sales. If he wants to earn more than \$2500 this month, what should Jack's total sales be?

Solution:

Let x be the total sales.

$$\text{Commission} = 10\% \text{ of } x$$
$$= \frac{10}{100}(x)$$

$$\text{Jack's salary} = 800 + \frac{10}{100}(x)$$

$$800 + \frac{10}{100}(x) > 2500 \quad \text{(Multiply every term by 10.)}$$
$$8000 + x > 25\,000$$
$$x > 25\,000 - 8000$$
$$x > 17\,000$$

Jack has to sell more than \$17 000 worth of merchandise.

Now look at another example.

Example 5

If five times a number is decreased by 5 and the result is divided by -2, the quotient is greater than 15. Find the number.

Solution:

Let x be the number.

Five time the number can be expressed as $5x$.
When this is decreased by 5, you get $5x - 5$.
Then the result is divided -2.
Thus, $\frac{5x-5}{-2}$ represents the quotient.

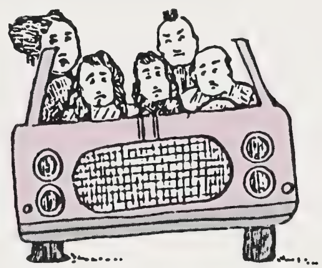
$$\frac{5x-5}{-2} > 15$$
$$\frac{(5x-5)}{(-2)}(-2) < (15)(-2) \quad \text{(Multiply both sides by } -2 \text{ and reverse the inequality sign.)}$$
$$5x - 5 < -30 \quad \text{(Add 5 to both sides.)}$$
$$5x < -25$$
$$\frac{5x}{5} < \frac{-25}{5} \quad \text{(Divide both sides by 5.)}$$
$$x < -5$$

The number is any number less than 5.

Now do the following questions to help you understand inequalities.

Do questions 1 and 3. If you want more practice, do questions 2 and 4.

1. Muhammed calculates that the total mass of five passengers plus 150 kg of luggage will exceed the 600 kg capacity of his little car. Find the average mass of each passenger.



2. The difference of two numbers is 4. Two times the smaller number plus the larger number is less than or equal to 25. Find a range of possible values for the smaller number.

3. Five greater than three times a certain number is less than 14. Determine a range of possible values for the number.

4. How much water must be added to 2 L of a 30% alcohol solution to obtain a mixture that is less than a 10% alcohol solution?

For solutions to **Activity 2**, turn to the **Appendix**, **Topic 2**.

Note: Five greater in this situation means you should add 5. When it says 5 is greater than it means 5 >.

If you require help, do the Extra Help section.

If you want more challenging explorations, do the Extensions section.

$\Big\}$ You may decide to do both.

Extra Help

The following questions review the basic skills needed to solve inequalities. Fill in the blanks, answer short questions, and check your answers as you do these questions.

1. Solve and graph $x-2>5$.

   Solution:

   What is your first step?

   $$x-2+(\quad)>5+(\quad)$$
   $$x>(\quad)$$

   Graph:

   In your graph, did you use a solid dot or an open (hollow) dot? Why?

2. Solve and graph $x+1\le 3$.

   Solution:

   What is your first step?

   $$x+1-(\quad)\le 3-(\quad)$$
   $$x\le(\quad)$$

   Graph:

   In your graph, did you use a solid dot or an open (hollow) dot? Why?

3. Solve and graph $\frac{x}{3} \geq 4$.

Solution:

What is your first step?

$$\frac{x}{3}(\quad) \geq 4(\quad)$$

$$x \geq (\quad)$$

Did you reverse the inequality sign? Why?

Graph:

4. Solve and graph $\frac{x}{-5} \leq -3$.

Solution:

What is your first step?

Insert the inequality sign and fill in the blanks.

$$\frac{x}{(-5)}(\quad) \quad (-3)(\quad)$$

$$x \quad (\quad)$$

Did you reverse the inequality sign? Why?

Graph:

5. Solve and graph $3x > -1$.

Solution:

What is your first step?

Insert inequality signs and fill in the blanks.

$$\frac{3x}{(\quad)} \quad \frac{-1}{(\quad)}$$

$$x \quad (\quad)$$

Did you reverse the inequality sign? Why?

Graph:

6. Solve and graph $-3x < 6$.

Solution:

What is your first step?

Insert inequality signs and fill in the blanks.

$(-3x) \quad 6$

$(\quad) \quad (\quad)$

$x \quad (\quad)$

Did you reverse the inequality sign? Why?

Graph:

For solutions to **Extra Help**, turn to the **Appendix, Topic 2**.

If you find the word problems difficult, try to follow the four step problem-solving procedure as shown.

Example 6

There are two consecutive numbers. The sum of the two numbers is divided by 3 and the quotient is less than 7. Find the smaller number.

Solution:

Step 1: Understand the problem.
Let x be the smaller number.
Then $x+1$ is the larger number.

Step 2: Develop a plan.

$$\frac{x+(x+1)}{3} < 7$$

Step 3: Carry out the plan.

$$\frac{x+(x+1)}{3}(3) < 7(3) \quad \text{(Multiply both sides by 3.)}$$

$$2x+1 < 21 \quad \text{(Simplify.)}$$

$$2x+1-1 < 21-1 \quad \text{(Subtract 1 from both sides.)}$$

$$2x < 20$$

$$\frac{2x}{(2)} < \frac{20}{(2)} \quad \text{(Divide both sides by 2.)}$$

$$x < 10$$

Step 4: Look back.

Substitute $x = 9$ in the inequality.

LS	RS
$\frac{x+(x+1)}{3}$	7
$\frac{(9)+[(9)+1]}{3}$	7
$\frac{19}{3}$	7
$6\frac{1}{3}$	7

LS < RS

Therefore, the inequality is true.

Substitute $x = 11$ in the inequality.

LS	RS
$\frac{x+(x+1)}{3}$	7
$\frac{(11)+[(11)+1]}{3}$	7
$\frac{23}{3}$	7
$7\frac{2}{3}$	7

LS > RS

Therefore, the inequality is **not** true.

These tests suggest that the inequality is true for $x < 10$. Therefore, the smaller number is less than 10.

Now you should be ready to do the following questions.

Do either part a or part b of questions 7 through 12. If you want more practice, go back and do the other parts. Then do question 13.

7. Solve and graph.

   a. $x - 7 \leq 2$

   b. $x - 3 > -2$

8. Solve and graph.

   a. $x + 1 < 3$

   b. $x + 2 \geq -3$

9. Solve and graph.

   a. $\frac{x}{5} \geq \frac{1}{5}$

   b. $\frac{x}{2} \leq \frac{1}{3}$

10. Solve and graph.

a. $\frac{-x}{18} \le \frac{1}{6}$

b. $\frac{x}{21} > \frac{-1}{3}$

11. Solve and graph.

a. $3x+1>7$

b. $5x+3<7$

12. Solve and graph.

a. $-2x-1<-11$

b. $-3x+2>-10$

13. The sum of two consecutive even integers is greater than 38. Find the smaller integer.

For solutions to **Extra Help**, turn to the **Appendix, Topic 2.**

After you finish these questions, you should review the examples provided in **Topic 2.**

Extensions

You have learned some of the basic principles of inequalities. Can you prove that $ac > bd$ if $a > b > 0$ and $c > d > 0$? Before you try to prove this, look at the following example.

Example 7

Given that $a > b$ and $c > d$, prove that $a+c > b+d$.

Solution:

Since $a > b$, it follows that $a-b>0$.
Since $c > d$, it follows that $c-d>0$.
Since $(a-b)$ and $(c-d)$ are positive, $(a-b)+(c-d)$ is positive.

$$
\begin{aligned}
(a-b)+(c-d)&>0\\
(a+c)-b-d&>0\\
(a+c)-(b+d)&>0\\
(a+c)-(b+d)+(b+d)&>(b+d)\\
(a+c)&>(b+d)
\end{aligned}
$$

Now try to prove the other principle.

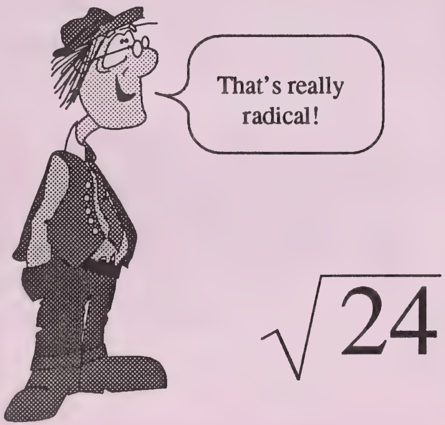
Prove that $ac > bd$ if $a > b > 0$ and $c > d > 0$.

For solutions to **Extensions**, turn to the **Appendix, Topic 2.**

Topic 3 Simple Quadratic and Radical Equations

Introduction

You have worked with linear equations in one variable. Many equations, however, are not linear. For example, if the area of a square is given, and you want to find the dimensions of the square, you have to solve an equation of degree 2, which is called a quadratic equation. In this topic you are going to look at some simple quadratic equations. After that, you will also look at equations involving radicals.



What Lies Ahead

Throughout this topic you will learn to

1. solve and verify simple quadratic equations which are easily reducible to $x^2 = a$
2. solve and verify simple radical equations which are easily reducible to $\sqrt{x} = b$

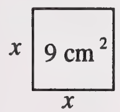
Now that you know what to expect, turn the page to begin your study of simple quadratic and radical equations.

Exploring Topic 3

Activity 1

Solve and verify simple quadratic equations which are easily reducible to $x^2 = a$.

If you know the area of a square, how do you find the dimensions of the square? For example, if the area of a square is 9 cm^2, how do you find the length of the side x? Since $x \bullet x = x^2$ is the area of a square, you have an equation $x^2 = 9$.



Now, if you want to find x, you have to solve this equation. Do you remember that $\sqrt{\ }$ **undoes** squaring? In order to find x, take the square root of both sides.

$$\sqrt{x^2} = \pm\sqrt{9}$$
$$x = \pm 3$$

Verify:

When $x = 3$:

LS	RS
x^2	9
$(3)^2$	9
9	9

LS = RS

When $x = -3$:

LS	RS
$(-3)^2$	9
9	9

LS = RS

Therefore, $x = 3$ or $x = -3$.

Since the length of a side cannot be negative, only $+3$ is accepted as a solution. Note that the number under the square root sign in the equation must not be negative because the square root of a negative number is undefined in the set of real numbers. If the question is not a word problem, make sure that you have one positive solution and one negative solution when you take the square root of a number.

An equation such as $x^2 = 9$ is called a quadratic equation because the highest degree is 2. A quadratic equation without a first degree term is the simplest type of quadratic equation. You can solve this kind of quadratic equation by leaving only the x^2 on the left side of the equation and moving all the constants to the right side. Then you can take the square root of both sides.

Note: You must consider both the positive and the negative solutions when taking the square root of both sides of an equation.

Both $(3)^2 = 9$ and $(-3)^2 = 9$. Therefore, $\pm\sqrt{9} = +3$ or -3.

$x \bullet x = x^2$

Remember, x^2 is **not** $2x$.

Note that $\sqrt{x^2} = x$ because $\sqrt{n}$ involves finding a number which when multiplied by itself is equal to n. Thus, since $x \bullet x = x^2$, it follows that $\sqrt{x^2} = x$.

Example 1

Solve $3x^2 = 12$.

Solution:

$$\frac{3x^2}{3} = \frac{12}{3} \quad \text{(Divide both sides by 3.)}$$

$$x^2 = 4$$

$$\sqrt{x^2} = \pm\sqrt{4} \quad \text{(Take the square root of both sides.)}$$

$$x = \pm 2$$

Verify:

When $x = 2$:

LS	RS
$3(2)^2$	12
$3(4)$	12
12	12

LS = RS

When $x = -2$:

LS	RS
$3(-2)^2$	12
$3(4)$	12
12	12

LS = RS

If the constant on the right side is a fraction you have to take the square root of both the numerator and the denominator.

Example 2

Solve $16x^2 = 49$.

Solution:

$$\frac{16x^2}{16} = \frac{49}{16}$$

$$x^2 = \frac{49}{16}$$

$$\sqrt{x^2} = \pm\sqrt{\frac{49}{16}}$$

$$x = \pm\frac{7}{4}$$

Verify:

When $x = \frac{7}{4}$:

LS	RS
$16\left(\frac{7}{4}\right)^2$	49
$16\left(\frac{49}{16}\right)$	49
49	49

LS = RS

Recall: The **BEDMAS** rule tells you to simplify exponents before multiplying.

Therefore, $\left(\frac{7}{4}\right)^2 = \frac{49}{16}$ before multiplying by 16.

When $x = -\frac{7}{4}$:

LS	RS
$16\left(\frac{-7}{4}\right)^2$	49
$16\left(\frac{49}{16}\right)$	49
49	49
LS =	RS

The next example involves two constant terms.

Example 3

Solve $5x^2 - 7 = 13$.

Solution:

$$
\begin{aligned}
5x^2 - 7 + 7 &= 13 + 7 && \text{(Add 7 to both sides.)} \\
5x^2 &= 20 \\
\frac{5x^2}{5} &= \frac{20}{5} && \text{(Divide both sides by 5.)} \\
x^2 &= 4 \\
\sqrt{x^2} &= \pm\sqrt{4} \\
x &= \pm 2
\end{aligned}
$$

Verify:

When $x = 2$:

LS	RS
$5(2)^2 - 7$	13
$5(4) - 7$	13
$20 - 7$	13
13	13
LS =	RS

When $x = -2$:

LS	RS
$5(-2)^2 - 7$	13
$5(4) - 7$	13
$20 - 7$	13
13	13
LS =	RS

Study the following example to see what to do if the second-degree term involves more than just a variable.

Example 4

Solve $(5x - 3)^2 = 144$.

Solution:

$$
\begin{aligned}
\sqrt{(5x-3)^2} &= \pm\sqrt{144} \\
5x - 3 &= \pm 12
\end{aligned}
$$

Since 12 can be positive or negative, there are two different cases.

Case 1

$$5x - 3 = 12$$
$$5x = 15$$
$$x = 3$$

Verify:

When $x = 3$:

LS	RS
$(5x-3)^2$	144
$[5(3)-3]^2$	144
$(15-3)^2$	144
$(12)^2$	144
144	144

LS = RS

Case 2

$$5x - 3 = -12$$
$$5x = -9$$
$$x = -\frac{9}{5}$$

Verify:

When $x = \frac{-9}{5}$:

LS	RS
$(5x-3)^2$	144
$\left[5\left(\frac{-9}{5}\right)-3\right]^2$	144
$(-9-3)^2$	144
$(-12)^2$	144
144	144

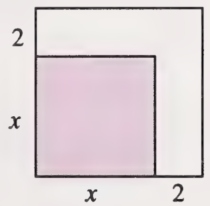
LS = RS

Therefore, the solutions are $x = 3$ or $x = -\frac{9}{5}$.

Now follow the solution to a word problem.

Example 5

Refer to the following diagram. If the area of the larger square is 9 cm^2 find the area of the small square.



Solution:

The length of one side of the large square is $x+2$.

$$(x+2)^2 = 9$$
$$\sqrt{(x+2)^2} = \pm\sqrt{9}$$
$$(x+2) = \pm 3$$

Case 1

$$x+2 = 3$$
$$x = 1$$

Case 2

$$x+2 = -3$$
$$x = -5$$

Discard Case 2 because x cannot be negative.

$$\text{Area of small square} = 1\text{ cm} \times 1\text{ cm}$$
$$= 1\text{ cm}^2$$

The area of the small square is 1 cm^2.

Now try the following questions.

Do either part a or part b of questions 1 through 4. Then do question 5. If you require more practice, go back and do the other parts.

1. Solve and verify.

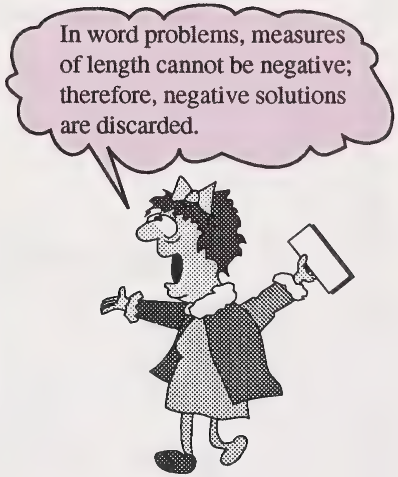
   a. $x^2 = 49$

   b. $x^2 = 121$

2. Solve and verify.

   a. $5x^2 = 45$

   b. $7a^2 = 252$



3. Solve and verify.

   a. $7x^2 - 3 = 60$

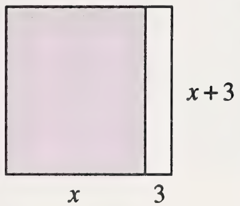
   b. $9x^2 - 2 = 47$

4. Solve and verify.

   a. $(3x-1)^2 = 196$

   b. $(5x+2)^2 = 1369$

5. Refer to the following figure. If the area of the larger square is 121 cm^2, find the area of the shaded rectangle.



For solutions to **Activity 1**, turn to the **Appendix**, **Topic 3**.

Activity 2

Solve and verify simple radical equations which are easily reducible to $\sqrt{x} = b$.

You have learned how to solve a simple quadratic equation of the form $x^2 = a$ by taking the square root of both sides. There is another kind of equation which involves a radical. They are called radical equations. To solve a radical equation, you have to square both sides.

Study the following example.

Example 6

Solve $\sqrt{x} = 3$.

Solution:

Square both sides.

$$\left(\sqrt{x}\right)^2 = 3^2$$
$$x = 9$$

The solution seems to be 9. Now you must verify it.

Verify: Substitute 9 into the original equation.

LS	RS
$\sqrt{9}$	3
3	3

LS = RS

Therefore, 9 is the solution.

Now look at another example.

Example 7

Solve $2\sqrt{x}+9=1$.

Solution:

$$\begin{aligned} 2\sqrt{x}+9-9&=1-9 && \text{(Subtract 9 from both sides.)} \\ 2\sqrt{x}&=-8 && \text{(Divide both sides by 2.)} \\ \sqrt{x}&=-4 \\ x&=(-4)^2 && \text{(Square both sides.)} \\ x&=16 \end{aligned}$$

The solution seems to be 16.

Verify:

LS	RS
$2\left(\sqrt{16}\right)+9$	1
$2(4)+9$	1
$8+9$	1
17	1

LS $\neq$ RS

Thus, 16 is not a real solution.

What is wrong in the solution to the preceding example? It is the technique you have used in solving radical equations. After squaring both sides, you have a new equation. The new equation may or may not be equivalent to the original equation.

For example,

$2 \neq -2$

However, if you square both sides, their squares are equal.

$$2^2=(-2)^2$$
$$4=4$$

Therefore, you must check your solution. If it does not satisfy the original equation, you have to discard it. In Example 7 the solution does not check. Therefore, you say there is no real solution.

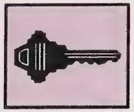
Note that $\sqrt{16}=4$, not ± 4, because $\sqrt{16}$ means the positive square root of 16. There is no negative sign in front of the radical sign.

Hoewever, $x^2=16$

$$\begin{aligned} x&=\pm\sqrt{16} \\ &=\pm 4 \end{aligned}$$

If you write $\sqrt{16}$, it means that only the positive root is chosen.

The solution $x=16$ is called an **extraneous root**. An extraneous root is a solution that does not check.



Remember: You must verify your answer.

Look at another example that has more than just a variable in the radical.

Example 8

Solve $\sqrt{3x+1}=4$.

Solution:

$$\left(\sqrt{3x+1}\right)^2=(4)^2 \quad \text{(Square both sides.)}$$
$$3x+1=16 \quad \text{(Subtract 1 from both sides.)}$$
$$3x=15 \quad \text{(Divide both sides by 3.)}$$
$$x=5$$

Verify:

LS	RS
$\sqrt{3(5)+1}$	4
$\sqrt{15+1}$	4
$\sqrt{16}$	4
4	4

LS = RS

The solution is $x=5$.

What are some real-life applications of radical equations? Look at the following word problem.

Example 9

When an automobile is moving in a circular path, the velocity of the car (v) is the square root of the product of the acceleration of the car (a) and the radius of the curve (r). The formula is $v=\sqrt{ar}$. If the velocity of the car (v) is 24 m/s and the radius of the curve (r) is 16, find the acceleration in m/s^2.

Solution:

$$v=\sqrt{ar}$$
$$\sqrt{ar}=v$$
$$\sqrt{16a}=24 \quad \text{(Square both sides.)}$$
$$16a=576$$
$$a=\frac{576}{16}$$
$$=36\ \text{m}/\text{s}^2$$

Use your calculator to find the square of a number or the square root of a number. If you need information on how to use your calculator, go to the **Extra Help** section.

Verify:

LS	RS
$\sqrt{16(36)}$	24
$\sqrt{576}$	24
24	24
LS =	RS

Therefore, $a = 36\ \text{m/s}^2$ is a solution.

The acceleration of the car is $36\ \text{m/s}^2$.

Now try some questions.

Do either part a or part b of questions 1 through 4. Then do question 5.

1. Solve and verify.

   a. $3\sqrt{y} = 147$

   b. $5\sqrt{x} = 320$

2. Solve and verify.

   a. $2\sqrt{x} - 3 = 19$

   b. $7\sqrt{x} - 5 = 23$

3. Solve and verify.

   a. $\sqrt{3x-2} = 5$

   b. $\sqrt{6x-3} = 3$

4. Solve and verify.

   a. $3\sqrt{x} + 5 = 2$

   b. $8\sqrt{x} + 46 = 6$

5. The period (t) of a pendulum depends on the length of the pendulum in metres. The formula is $t = 2\pi\sqrt{\frac{l}{9.8}}$, where $\pi = 3.1416$.

   If the period of the pendulum is four seconds, find the length of the pendulum. Round your answer to the nearest hundredth of a metre. Use your calculator here.

For solutions to **Activity 2**, turn to the **Appendix, Topic 3**.

The **period** is the time taken for a complete vibration.

The **frequency** is the number of complete vibrations per unit of time.

If you require help, do the Extra Help section.
If you want more challenging explorations, do the Extensions section.
} You may decide to do both.

Extra Help

The following shows you how to use your calculator to find the square root of a number.

- If your calculator has a $\boxed{\sqrt{\ }}$ key, follow these steps:

  Step 1: Enter the number.

  Step 2: Press the $\boxed{\sqrt{\ }}$ key.

- If your calculator has an $\boxed{x^2}$ key, follow these steps:

  Step 1: Enter the number.

  Step 2: Press the $\boxed{\text{INV}}$ key.

  Step 3: Press the $\boxed{x^2}$ key.

The following shows you how to use your calculator to find the square of a number.

- If your calculator has an $\boxed{x^2}$ key, follow these steps:

  Step 1: Enter the number.

  Step 2: Press the $\boxed{x^2}$ key.

- If your calculator has a $\boxed{\sqrt{\ }}$ key, follow these steps:

  Step 1: Enter the number.

  Step 2: Press the $\boxed{\text{INV}}$ key.

  Step 3: Press the $\boxed{\sqrt{\ }}$ key.

- If you calculator does not have an $\boxed{x^2}$ key or a $\boxed{\sqrt{\ }}$ key, follow these steps:

  Step 1: Enter the number.

  Step 2: Press the $\boxed{\times}$ key.

  Step 3: Press the $\boxed{=}$ key.

Do either part a or part b of the following questions. If you want more practice, go back and do the other parts.

1. Calculate the following.

   a. $\sqrt{3136}$

   b. $\sqrt{15\,376}$

2. Calculate the following.

   a. $\sqrt{28.09}$

   b. $\sqrt{0.133\,225}$

3. Calculate the following.

   a. $(57.2)^2$

   b. 63^2

4. Calculate the following.

   a. $(0.21)^2$

   b. 0.53^2

For solutions to **Extra Help**, turn to the **Appendix, Topic 3**.

Extensions

You know how to use a calculator to find the square root of a number. Do you know how to calculate it without using a calculator? The following examples show you a method.

Example 10

Find the square root of 5476.

Solution:

Step 1: Starting from a decimal point, separate the number into groups of two digits. Place a decimal point directly above the original decimal point.

$$\begin{array}{|c|c}
 & \quad . \\ \hline
54 & 76\ . \\
 &
\end{array}$$

Step 2: The closest perfect square smaller than 54 is 49. The square root of 49 is 7. Place 7 directly above the digit 4 and place 49 under 54.

$$\begin{array}{|c|c}
\boxed{7} & \quad . \\ \hline
54 & 76\ . \\
\boxed{49} &
\end{array}$$

Step 3: Subtract 49 from 54 and bring down the next pair of digits, the 76.

$$\begin{array}{|c|c}
7 & \quad . \\ \hline
54 & 76\ . \\
49 & \\ \hline
\boxed{5\ \ 76} &
\end{array}$$

Step 4: Multiply the quotient 7 by 20 (always 20) and place the product beside the new dividend 576.

$$\begin{array}{r|l|l} & 7 & . \\ \hline & 54 & 76\ . \\ & 49 & \\ \hline \boxed{7\times 20=140} & 5 & 76 \end{array}$$

Step 5: Use 140 as a trial divisor. It looks like 140 will divide into 576 four times. Add this digit to 140.

$$\begin{array}{r} 140 \\ +\ 4 \\ \hline 144 \end{array}$$

Step 6: Place 4 directly above the digit 6 of the original number. Multiply 144 by 4 and place the product under 576.

$$\begin{array}{r|l|l} & 7 & \boxed{4}. \\ \hline & 54 & 76\ . \\ & 49 & \\ \hline & 5 & 76 \\ \boxed{4\times 144} & \boxed{5} & \boxed{76} \\ \hline & & 0 \end{array}$$

Step 7: Subtract. In this case the remainder is zero.

Step 8: Thus, the square root of 5476 is 74.

The next example involves the square root of a decimal numeral.

Example 11

Find $\sqrt{151.29}$.

Solution:

Step 1: Starting from the decimal point, separate the number into groups of two digits. Place a decimal point directly above the original one.

$$\begin{array}{|l|l|l} & & . \\ \hline 1 & 51 & .29 \end{array}$$

Step 2: The perfect square closest to the first group (which is 1) is 1. The square root of 1 is 1. Place 1 directly above the first digit 1. Also place the product of $1\times 1=1$ under 1.

$$\begin{array}{|l|l|l} 1 & & . \\ \hline 1 & 51 & .29 \\ 1 & & \\ \hline & & \end{array}$$

Step 3: Subtract 1 from 1. Bring down the next pair of two digits.

$$\begin{array}{|l|l|l} 1 & & . \\ \hline 1 & 51 & .29 \\ 1 & & \\ \hline & \boxed{51} & \end{array}$$

Step 4: Multiply the quotient 1 by 20 and place the product beside the new dividend 51.

$$\begin{array}{r|l|l|l} & 1 & & . \\ \hline & 1 & 51 & .29 \\ & 1 & & \\ \hline \boxed{1\times 20=20} & & 51 & \end{array}$$

Step 5: Use 20 as a trial divisor. Check how many times this 20 will go into the 51. Add the digit you want to try to 20. In this case the digit is 2.

$$\begin{array}{r} 20 \\ +\ 2 \\ \hline 22 \end{array}$$

Step 6: Place 2 directly above the group 51. Multiply 22 by 2 and place the product under 51.

	1	$\boxed{2}$	.
	1	51	. 29
	1		
22		51	
		$\boxed{44}$	

Step 7: Subtract 44 from 51 and bring down the next pair 29.

	1	2	.
	1	51	. 29
	1		
22		51	
		44	
		$\boxed{7}$	$\boxed{29}$

Step 8: Multiply the quotient 12 by 20 (always 20) and place it beside the dividend 729.

	1	2	.
	1	51	. 29
	1		
22		51	
		44	
$\boxed{12 \times 20 = 240}$		$\boxed{7}$	$\boxed{29}$

Step 9: Use $12 \times 20 = 240$ as a trial divisor. How many times will 240 go into 729? Add the digit you want to try to 240. In this case, the digit is 3.

$$\begin{array}{r} 240 \\ +\ 3 \\ \hline 243 \end{array}$$

Step 10: Place 3 directly above the last group (29). Multiply 243 by 3 and place the product under 729.

	1	2	. $\boxed{3}$
	1	51	. 29
	1		
22		51	
		44	
243		7	29
		$\boxed{7}$	$\boxed{29}$
			0

Step 11: Subtract. The remainder is 0.

Step 12: Therefore, the square root of 151.29 is 12.3.

You should now appreciate what a calculator can do for you. Now try to see if you can do at least one of these on your own.

1. Find $\sqrt{1681}$.

2. Find $\sqrt{1049.76}$.

For solutions to **Extensions**, turn to the **Appendix**, **Topic 3**.

Unit Summary

What You Have Learned

Here are the key ideas you have learned in this unit.

- To solve an equation, isolate the variable and add like terms.
- To solve an equation with parentheses, remember to remove the parentheses first; then add like terms.
- To translate an English sentence into algebra, use a variable. For example, two consecutive integers can be shown as x and $x+1$.
- To solve an equation with rational coefficients, multiply every term by a common denominator.
- The symbol $>$ means greater than.
- The symbol $<$ means less than.
- The symbol $\geq$ means greater than or equal to.
- The symbol $\leq$ means less than or equal to.
- Multiplying or dividing by a negative number will reverse the inequality sign.
- In graphs, a closed dot includes the solution, while an open dot does not include the solution.
- A linear equation has a degree of 1.
- A quadratic equation has a degree of 2.
- Finding the square root of a number gives you two possible solutions: a positive one and a negative one.
- To solve a quadratic equation of the form $x^2 = a$, take the square root of both sides.

Unit Summary

- To solve a radical equation, first square both sides.

  For example,

$$\begin{aligned} \sqrt{x+1} &= 4 \\ \left(\sqrt{x+1}\right)^2 &= (4)^2 \\ x+1 &= 16 \\ x &= 15 \end{aligned}$$

  Then, verify the solution in the original equation.

  Watch for extraneous roots.

You are now ready to complete the **Unit Assignment**.

Appendix

Solutions

Review

Topic 1 Solving and Verifying Linear Equations

Topic 2 Inequalities

Topic 3 Simple Quadratic and Radical Equations

Review

1. $x+3=8$

   $x+3-3=8-3$

   $x=5$

2. $x-7=3$

   $x-7+7=3+7$

   $x=10$

3. $3x=18$

   $\frac{3x}{3}=\frac{18}{3}$

   $x=6$

4. $\frac{x}{4}=2$

   $\frac{x}{4}(4)=(2)(4)$

   $x=8$

5. $3x-3=x+5$

   $3x-3+3=x+5+3$

   $3x=x+8$

   $3x-x=x+8-x$

   $2x=8$

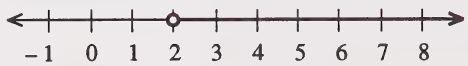
   $x=4$

6. $x+1>3$

   $x+1-1>3-1$

   $x>2$

   Graph:

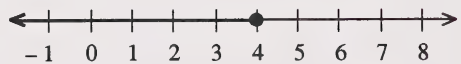


7. $x-3\le 1$

   $x-3+3\le 1+3$

   $x\le 4$

   Graph:



8. $\frac{x}{3} < 1$

$\frac{x}{3}(3) < (1)(3)$

$x < 3$

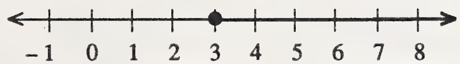
Graph:

9. $5x \geq 15$

$\frac{5x}{(5)} \geq \frac{15}{(5)}$

$x \geq 3$

Graph:



If you had trouble with the **Review**, go to **Mathematics 9**.

Exploring Topic 1

Activity 1

Translate English sentences into algebra.

1. a. $x+3$ b. $x-7$

 c. $\frac{8}{x}$ d. $6x$

 e. $5-x$ f. $42t$ cents

 g. $2x-1$ h. $\frac{1}{2}n+3$ or $\frac{n}{2}+3$

 i. $\frac{n}{3}$ or $n \div 3$ j. $2x+5$

2. a. $x =$ Sonja's age
 $2x =$ Jim's age

 b. $x =$ Kurtis' earnings
 $x+300 =$ Nadia's earnings

 c. x cm $=$ width
 $(x+5)$ cm $=$ length

 d. $x =$ number of girls
 $2x-4 =$ number of boys

Activity 2

Solve and verify simple linear equations with integral coefficients.

1. a. $3x-2(x+4)=5-3(x-1)$

$$\begin{aligned} 3x-2x-8&=5-3x+3 && \text{(Simplify.)}\\ x-8&=8-3x && \text{(Add 8 to both sides.)}\\ x-8+8&=8-3x+8 && \text{(Simplify.)}\\ x&=16-3x && \text{(Add } 3x \text{ to both sides.)}\\ x+3x&=16-3x+3x\\ 4x&=16 && \text{(Divide by 4.)}\\ \frac{4x}{4}&=\frac{16}{4}\\ x&=4 \end{aligned}$$

Verify:

LS	RS
$3x-2(x+4)$	$5-3(x-1)$
$3(4)-2[(4)+4]$	$5-3[(4)-1]$
$12-2(8)$	$5-3(3)$
$12-16$	$5-9$
-4	-4

LS = RS

b. $1+3(x-5)=x-2(x+3)$

$$\begin{aligned} 1+3(x-5)&=x-2(x+3)\\ 1+3x-15&=x-2x-6\\ 3x-14&=-x-6\\ 3x-14+14&=-x-6+14\\ 3x&=-x+8\\ 3x+x&=-x+x+8\\ 4x&=8\\ \frac{4x}{4}&=\frac{8}{4}\\ x&=2 \end{aligned}$$

Verify:

LS	RS
$1+3(x-5)$	$x-2(x+3)$
$1+3[(2)-5]$	$(2)-2[(2)+3]$
$1+3(-3)$	$2-2(5)$
$1-9$	$2-10$
-8	-8

LS = RS

c. $x(x+1)+3=(x-1)(x+5)$ (Multiply the factors.)

$$x^2+x+3=x^2-x+5x-5$$
$$x^2+x+3=x^2+4x-5$$
$$x^2-x^2+x+3=x^2-x^2+4x-5 \quad \text{(Subtract } x^2 \text{ from both sides.)}$$
$$x+3=4x-5$$
$$x-4x+3-3=4x-4x-5-3 \quad \text{(Subtract } 4x \text{ and 3 from both sides.)}$$
$$-3x=-8$$
$$x=\frac{8}{3}$$

Verify:

LS	RS
$x(x+1)+3$	$(x-1)(x+5)$
$\frac{8}{3}\left(\frac{8}{3}+1\right)+3$	$\left(\frac{8}{3}-1\right)\left(\frac{8}{3}+5\right)$
$\frac{8}{3}\left(\frac{8}{3}+\frac{3}{3}\right)+3$	$\left(\frac{8}{3}-\frac{3}{3}\right)\left(\frac{8}{3}+\frac{15}{3}\right)$
$\frac{8}{3}\left(\frac{11}{3}\right)+3$	$\left(\frac{5}{3}\right)\left(\frac{23}{3}\right)$
$\frac{88}{9}+3$	$\frac{115}{9}$
$\frac{88}{9}+\frac{27}{9}$	$\frac{115}{9}$
$\frac{115}{9}$	$\frac{115}{9}$

LS = RS

Therefore, $x=\frac{8}{3}$ is a solution to the equation.

d. $(9x+1)(x-2)=(3x-1)^2$

$$(9x+1)(x-2)=(3x-1)(3x-1) \quad \text{(Multiply the factors.)}$$
$$9x^2+x-18x-2=9x^2-3x-3x+1$$
$$9x^2-17x-2=9x^2-6x+1$$
$$9x^2-9x^2-17x-2=9x^2-9x^2-6x+1 \quad \text{(Subtract } 9x^2 \text{ from both sides.)}$$
$$-17x-2=-6x+1$$
$$-17x+6x-2+2=-6x+6x+1+2 \quad \text{(Add } 6x \text{ and 2 to both sides.)}$$
$$-11x=3$$
$$x=-\frac{3}{11}$$

Verify:

LS	RS
$(9x+1)(x-2)$	$(3x-1)^2$
$\left[9\left(\frac{-3}{11}\right)+1\right]\left(\frac{-3}{11}-2\right)$	$\left[3\left(\frac{-3}{11}\right)-1\right]^2$
$\left(\frac{-27}{11}+1\right)\left(\frac{-3}{11}-2\right)$	$\left(\frac{-9}{11}-1\right)^2$
$\left(\frac{-27}{11}+\frac{11}{11}\right)\left(\frac{-3}{11}-\frac{22}{11}\right)$	$\left(\frac{-9}{11}-\frac{11}{11}\right)^2$
$\left(\frac{-16}{11}\right)\left(\frac{-25}{11}\right)$	$\left(\frac{-20}{11}\right)^2$
$\frac{400}{11}$	$\frac{400}{11}$

LS = RS

Therefore, $x=\frac{-3}{11}$ is a solution to the equation.

2. Let x be the smaller number.
Then $2x+8$ is the larger number.

$$\begin{aligned} \text{larger number} - \text{smaller number} &= 15 \\ (2x+8)-x &= 15 \\ 2x+8-x &= 15 \\ x+8 &= 15 \\ x &= 7 \end{aligned}$$

$$\begin{aligned} 2x+8 &= 2\times7+8 \\ &= 22 \end{aligned}$$

The two numbers are 7 and 22.

3. Let x be the son's age.
Then $3x+8$ is the mother's age.
Four years ago: Son's age $= x-4$

$$\begin{aligned} \text{Mother's age} &= 3x+8-4 \\ &= 3x+4 \end{aligned}$$

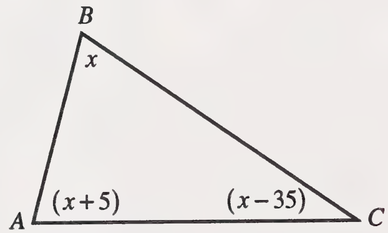
Mother's age four years ago $= 11 \times$ son's age four years ago

$$\begin{aligned} 3x+4 &= 11(x-4) \\ 3x+4 &= 11x-44 \\ 44+4 &= 11x-3x \\ 48 &= 8x \\ x &= \frac{48}{8} \\ x &= 6 \end{aligned}$$

$$\begin{aligned} 3x+8 &= 3\times6+8 \\ &= 26 \end{aligned}$$

Therefore, the mother's age is 26 and the son's age is 6.

4. Let x be the measure of $\angle B$.
Then $x+5$ is the measure of $\angle A$ and $x-35$ is the measure of $\angle C$.



$$\begin{aligned} (x)+(x+5)+(x-35) &= 180 \\ x+x+5+x-35 &= 180 \\ 3x-30 &= 180 \\ 3x &= 210 \\ x &= 70^\circ \\ x+5 &= 75^\circ \\ x-35 &= 35^\circ \end{aligned}$$

$\therefore \angle A = 75^\circ \quad \angle B = 70^\circ \quad \angle C = 35^\circ$

5. Let x be the width.
Then $x+5$ is the length.

$$\begin{aligned} 2(x)+2(x+5) &= 90 \\ 2x+2x+10 &= 90 \\ 4x &= 80 \\ x &= 20 \quad \text{(width)} \end{aligned}$$

$x+5=25$ (length)

Therefore, the dimensions are $20 \text{ cm} \times 25 \text{ cm}$.

6. Let x be the number of nickels.
Then $x+5$ is the number of dimes and $x-2$ is the number of quarters.

The value of x nickels is $5x$ ¢.
The value of $(x+5)$ dimes is $10(x+5)$ ¢.
The value of $(x-2)$ quarters is $25(x-2)$ ¢.

$$\begin{aligned} 5x+10(x+5)+25(x-2) &= 200 \\ 5x+10x+50+25x-50 &= 200 \\ 40x &= 200 \\ x &= 5 \quad \text{(nickels)} \end{aligned}$$

$$\begin{aligned} x+5 &= (5)+5 \\ &= 10 \quad \text{(dimes)} \end{aligned}$$

$$\begin{aligned} x-2 &= (5)-2 \\ &= 3 \quad \text{(quarters)} \end{aligned}$$

Therefore, there are five nickels, ten dimes, and three quarters.

Activity 3

Solve and verify simple linear equations with rational coefficients.

1. a. $\frac{5}{7}m = 8$

$$\overset{1}{\cancel{7}}\left(\frac{5}{\underset{1}{\cancel{7}}}m\right) = 7(8) \quad \text{(Multiply each term by 7.)}$$

$$5m = 56$$

$$\frac{5m}{5} = \frac{56}{5} \quad \text{(Divide both sides by 5.)}$$

$$m = \frac{56}{5}$$

$$= 11\frac{1}{5}$$

Verify:

LS	RS
$\frac{5}{7}m$	8
$\frac{5}{7}\left(11\frac{1}{5}\right)$	8
$\frac{5}{7}\times\frac{56}{5}$	8
8	8

LS = RS

b. $\frac{x}{3}+5=\frac{x}{2}-1$

$$\overset{2}{\cancel{6}}\left(\frac{x}{\underset{1}{\cancel{3}}}\right)+6(5)=\overset{3}{\cancel{6}}\left(\frac{x}{\underset{1}{\cancel{2}}}\right)-6(1)$$ (Multiply every term by 6.)

$$2x+30=3x-6$$ (Subtract $2x$ from both sides and add 6 to both sides.)

$$36=x$$

$$x=36$$

Verify:

LS	RS
$\frac{x}{3}+5$	$\frac{x}{2}-1$
$\frac{(36)}{3}+5$	$\frac{(36)}{2}-1$
$12+5$	$18-1$
17	17

LS = RS

c. $\frac{(x-2)}{3}-\frac{(x+1)}{5}=4$

$$\overset{5}{\cancel{15}}\left(\frac{x-2}{\underset{1}{\cancel{3}}}\right)-\overset{3}{\cancel{15}}\left(\frac{(x+1)}{\underset{1}{\cancel{5}}}\right)=15(4)$$ (Multiply every term by 15.)

$$5(x-2)-3(x+1)=(15)(4)$$

$$5x-10-3x-3=60$$

$$2x-13=60$$

$$2x=73$$

$$x=\frac{73}{2}$$

$$=36\frac{1}{2}$$

Verify:

LS	RS
$\frac{x-2}{3}-\frac{x+1}{5}$	4
$\frac{36\frac{1}{2}-2}{3}-\frac{36\frac{1}{2}+1}{5}$	4
$\frac{34\frac{1}{2}}{3}-\frac{37\frac{1}{2}}{5}$	4
$\frac{\frac{69}{2}}{3}-\frac{\frac{75}{2}}{5}$	4
$\frac{\overset{23}{\cancel{69}}}{2}\times\frac{1}{\underset{1}{\cancel{3}}}-\frac{\overset{15}{\cancel{75}}}{2}\times\frac{1}{\underset{1}{\cancel{5}}}$	4
$\frac{23}{2}-\frac{15}{2}$	4
$\frac{8}{2}$	4
4	4

LS = RS

d. $$\frac{2x-3}{4}=\frac{x+5}{3}$$

$$\overset{3}{\cancel{12}}\left(\frac{2x-3}{\underset{1}{\cancel{4}}}\right)=\overset{4}{\cancel{12}}\left(\frac{x+5}{\underset{1}{\cancel{3}}}\right)$$ (Multiply every term by 12.)

$$6x-9=4x+20$$

$$2x=29$$

$$x=\frac{29}{2}$$

Verify:

LS	RS
$\frac{2x-3}{4}$	$\frac{x+5}{3}$
$\frac{2\left(\frac{29}{2}\right)-3}{4}$	$\frac{\frac{29}{2}+5}{3}$
$\frac{29-3}{4}$	$\frac{\frac{29}{2}+\frac{10}{2}}{3}$
$\frac{26}{4}$	$\frac{39}{2}\times\frac{1}{3}$
$\frac{13}{2}$	$\frac{13}{2}$

LS = RS

2. Let x be the amount of water required.
   Then $5+x$ is the volume of the resulting solution.

   Since 70% of the original solution is sulphuric acid, $\frac{70}{100}(5)$ L is acid.

   Twenty percent or $\frac{20}{100}(5+x)$ L of the new solution is acid.

   Since no acid was added or taken away,

   $$\left(\begin{array}{l}\text{volume of acid in}\\ \text{the first solution}\end{array}\right)=\left(\begin{array}{l}\text{volume of acid in}\\ \text{the second solution}\end{array}\right).$$

$$\therefore \frac{70}{100}(5) = \frac{20}{100}(5+x)$$

$$\frac{350}{100} = \frac{100+20x}{100} \qquad \text{(Use the distributive property.)}$$

$$\frac{350}{100} \times 100 = \frac{100+20x}{100} \times 100 \qquad \text{(Multiply both sides by 100.)}$$

$$350 = 100 + 20x$$

$$\frac{250}{20} = \frac{20x}{20}$$

$$12.5 = x$$

$$x = 12.5$$

Verify:

LS	RS
$\frac{70}{100}(5)$	$\frac{20}{100}(5+12.5)$
$\frac{70}{20}$	$\frac{20(17.5)}{100}$
3.5	3.5

LS = RS

Brenda will need 12.5 L of water.

3. Let x be the smaller number.
   Then $x+6$ is the larger number.

$$\frac{1}{3}x + \frac{1}{2}(x+6) = 33$$

$$\overset{2}{\cancel{(6)}}\frac{1}{\underset{1}{\cancel{3}}}x + \overset{3}{\cancel{(6)}}\frac{1}{\underset{1}{\cancel{2}}}(x+6) = (6)33 \qquad \text{(Multiply each term by 6.)}$$

$$2x + 3(x+6) = 198 \qquad \text{(Use the distributive property.)}$$

$$2x + 3x + 18 = 198$$

$$5x = 180$$

$$x = 36$$

$$x + 6 = 36 + 6$$
$$= 42$$

Verify:

LS	RS
$\frac{1}{3}(36) + \frac{1}{2}(36+6)$	33
$12 + \frac{1}{2}(42)$	33
$12 + 21$	33
33	33

LS = RS

Therefore, the two numbers are 36 and 42.

4. Let x be the smaller number.
Then $x+4$ is the larger number because the difference between the two numbers is 4.

Also, $\frac{x+4}{2}$ is half the larger number.

$$\frac{x+4}{2}+2x=12$$

$$\overset{1}{\cancel{(2)}}\frac{x+4}{\underset{1}{\cancel{2}}}+(2)2x=(2)12$$

$$x+4+4x=24$$

$$5x=20$$

$$x=4$$

$$x+4=4+4$$

$$=8$$

Verify:

LS	RS
$\frac{x+4}{2}+2x$	12
$\frac{(4)+4}{2}+2(4)$	12
$\frac{8}{2}+8$	12
$4+8$	12
12	12

LS = RS

Therefore, the two numbers are 4 and 8.

5. Let x be the amount of water required.
Then $1+x$ is the volume of the resulting solution.

A 15% solution means that $\frac{15}{100}$ of the original solution is acid $\left(\frac{15}{100}\times 1\right)$.

A 10% concentration means that $\frac{10}{100}$ of the new solution is acid $\left[\frac{10}{100}(1+x)\right]$.

$$\begin{pmatrix}\text{volume of acid in}\\\text{original solution}\end{pmatrix}=\begin{pmatrix}\text{volume of acid in}\\\text{new solution}\end{pmatrix}$$

$$\therefore \frac{15}{100}\times 1=\frac{10}{100}(1+x)$$

$$\frac{15}{100}=\frac{10+10x}{100} \quad \text{(Use the distributive property.)}$$

$$\overset{1}{\cancel{100}}\times\frac{15}{\underset{1}{\cancel{100}}}=\frac{10+10x}{\underset{1}{\cancel{100}}}\times\overset{1}{\cancel{100}} \quad \text{(Multiply both sides by 100.)}$$

$$15=10+10x$$

$$\frac{5}{10}=\frac{10x}{10}$$

$$0.5=x$$

$$x=0.5$$

Verify:

LS	RS
$\frac{15}{100}\times 1$	$\frac{10}{100}(1+0.5)$
$\frac{15}{100}$	$\frac{10}{100}(1.5)$
0.15	$\frac{15}{100}$
0.15	0.15

LS = RS

Therefore, Helmut will need 0.5 L of water.

Extra Help

1. $\begin{array}{r|rr} 5 & 10 & 5 \\ \hline & 2 & 1 \end{array}$

   $\text{L.C.M.} = 5\times 2\times 1$
   $= 10$

2. $\begin{array}{r|rrr} 2 & 8 & 10 & 15 \\ \hline 5 & 4 & 5 & 15 \\ \hline & 4 & 1 & 3 \end{array}$

   $\text{L.C.M.} = 2\times 5\times 4\times 1\times 3$
   $= 120$

3. $\begin{array}{r|rrr} 2 & 8 & 12 & 18 \\ \hline 2 & 4 & 6 & 9 \\ \hline 3 & 2 & 3 & 9 \\ \hline & 2 & 1 & 3 \end{array}$

   $\text{L.C.M.} = 2\times 2\times 3\times 2\times 1\times 3$
   $= 72$

4. $\begin{array}{r|rrrr} 2 & 4 & 10 & 15 & 18 \\ \hline 5 & 2 & 5 & 15 & 9 \\ \hline 3 & 2 & 1 & 3 & 9 \\ \hline & 2 & 1 & 1 & 3 \end{array}$

   $\text{L.C.M.} = 2\times 5\times 3\times 2\times 1\times 1\times 3$
   $= 180$

Extensions

1. $3(x+k)-2(x-1)=k+2$

   Let $x=-4$.

   $$\begin{aligned} 3(-4+k)-2(-4-1) &= k+2 \\ -12+3k+10 &= k+2 \\ 3k-2 &= k+2 \\ 3k &= k+4 && \text{(Add 2 to both sides.)} \\ 2k &= 4 && \text{(Subtract } k \text{ from both sides.)} \\ k &= 2 \end{aligned}$$

   The value of k is 2.

2. $$\begin{aligned} \frac{5}{x}-\frac{1}{x} &= 2 \\ \frac{4}{x} &= 2 \\ 4 &= 2x && \text{(Multiply both sides by } x\text{.)} \\ x &= 2 \end{aligned}$$

Verify:

LS	RS
$\frac{5}{x}-\frac{1}{x}$	2
$\frac{5}{2}-\frac{1}{2}$	2
$\frac{4}{2}$	2
2	2

LS = RS

3. Let x be the daughter's age.
   Then $3x$ is Mr. Taylor's age.
   Ten years ago: Daughter's age $= x-10$
   Mr. Taylor's age $= 3x-10$

$$\begin{aligned}3x-10&=4(x-10)\\3x-10&=4x-40\\40-10&=4x-3x\\30&=x\\x&=30\end{aligned}$$

$$\begin{aligned}3x&=3(30)\\&=90\end{aligned}$$

Mr. Taylor is 90 years old.

4. The speed of one motorcycle is $\frac{2\text{ km}}{15\text{ s}}$.
   The speed of other motorcycle is $\frac{2\text{ km}}{20\text{ s}}$.

Let t be the elapsed when they pass each other.
(Because they started at the same time, they will have travelled for the same length of time when they meet.)

Distance travelled by one + distance travelled by other = 2 km
Distance = speed × time

$$\therefore \frac{2}{15}(t)+\frac{\overset{1}{\cancel{2}}}{\underset{10}{\cancel{20}}}(t)=2$$

$$\begin{aligned}4t+3t&=60 \quad \text{(Multiply both sides by 30.)}\\7t&=60\\t&=\frac{60}{7}\\&=8.57\end{aligned}$$

They will pass each other in 8.57 s.

5. Let x be the tens digit.
   Then $x-5$ is the units digit.

$$\begin{aligned}x+(x-5)&=9\\2x-5&=9\\2x&=14\\x&=7\end{aligned}$$

$$\begin{aligned}x-5&=(7)-5\\&=2\end{aligned}$$

The number is 72.

Exploring Topic 2

Activity 1

Solve, graph, and verify any linear inequalities which have rational coefficients, and apply the reverse-the-sign rule as needed.

1. a.

$$5x-7 \le 3x+11$$

$$5x-7-3x \le 3x+11-3x \quad \text{(Subtract } 3x \text{ from both sides.)}$$

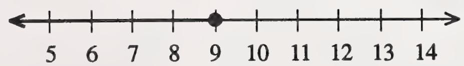
$$2x-7 \le 11$$

$$2x-7+7 \le 11+7 \quad \text{(Add 7 to both sides.)}$$

$$2x \le 18$$

$$x \le 9$$

Graph:



In the original inequality, LS $\le$ RS.

Verify:

Let $x = 7$, which is less than 9.

LS	RS
$5x-7$	$3x+11$
$5(7)-7$	$3(7)+11$
$35-7$	$21+11$
28	32

LS $<$ RS

Since $28 < 32$, the inequality is true when $x = 7$.

Let $x = 11$, which is greater than 9.

LS	RS
$5x-7$	$3x+11$
$5(11)-7$	$3(11)+11$
$55-7$	$33+11$
48	44

LS $>$ RS

Since 48 is not less than 44, the inequality is not true when $x = 11$.

These tests show that the inequality is true for $x \le 9$.

b. $3x+2 \geq 14-x$

$3x+2-2 \geq 14-x-2$ (Subtract 2 from both sides.)

$3x \geq 12-x$

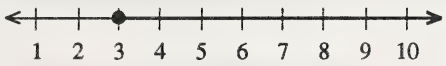
$3x+x \geq 12-x+x$ (Add x to both sides.)

$4x \geq 12$

$\frac{4x}{4} \geq \frac{12}{4}$ (Divide both sides by 4.)

$x \geq 3$

Graph:



In the original inequality, LS ≥ RS.

Verify:

Let $x=0$, which is less than 3.

LS	RS
$3x+2$	$14-x$
$3(0)+2$	$14-(0)$
$0+2$	14
2	14
LS <	RS

Since $2<14$, the inequality is not true when $x=0$.

Let $x=5$, which is greater than 3.

LS	RS
$3x+2$	$14-x$
$3(5)+2$	$14-(5)$
$15+2$	9
17	9
LS >	RS

Since $17>9$, the inequality is true when $x=5$.

These tests show that the inequality is true for $x \geq 3$.

2. a. $3(4-x) > x+8$

$12-3x > x+8$

$12-3x-12 > x+8-12$ (Subtract 12 from both sides.)

$-3x > x-4$

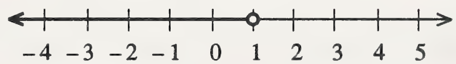
$-3x-x > x-4-x$ (Subtract x from both sides.)

$-4x > -4$

$\frac{-4x}{-4} < \frac{-4}{-4}$ (Divide both sides by -4 and reverse the inequality sign.)

$x<1$

Graph:



In the original inequality, LS > RS.

Verify:

Let $x = -3$, which is less than 1.

LS	RS
$3(4-x)$	$x+8$
$3[4-(-3)]$	$(-3)+8$
$3[4+3]$	5
$3(7)$	5
21	5

LS > RS

Since $21 > 5$, the inequality is true when $x = -3$.

Let $x = +3$, which is greater than 1.

LS	RS
$3(4-x)$	$x+8$
$3[4-(3)]$	$(3)+8$
$3(1)$	11
3	11

LS < RS

Since $3 < 11$, the inequality is not true when $x = 3$.

These tests show that the inequality is true for $x < 1$.

b. $2(2-x) \le 4(x-14)$

$4 - 2x \le 4x - 56$

$4 - 2x - 4 \le 4x - 56 - 4$ (Subtract 4 from both sides.)

$-2x \le 4x - 60$

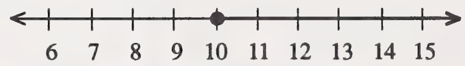
$-2x - 4x \le 4x - 60 - 4x$ (Subtract $4x$ from both sides.)

$-6x \le -60$

$\frac{-6x}{-6} \ge \frac{-60}{-6}$ (Divide both sides by -6 and reverse the inequality sign.)

$x \ge 10$

Graph:



In the original inequality, LS ≤ RS.

Verify:

Let $x = 12$, which is greater than 10.

LS	RS
$2(2-x)$	$4(x-14)$
$2[2-(12)]$	$4[(12)-14]$
$2(-10)$	$4(-2)$
-20	-8

LS < RS

Since $-20 < -8$, the inequality is true when $x = 12$.

Let $x = 8$, which is less than 10.

LS	RS
$2(2-x)$	$4(x-14)$
$2[2-(8)]$	$4[(8)-14]$
$2(-6)$	$4(-6)$
-12	-24

LS $>$ RS

Since $-12 > -24$, the inequality is not true when $x = 8$.

These tests show that the inequality is true for $x \geq 10$.

3. a.

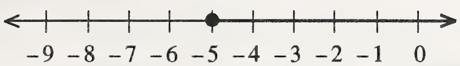
$$\frac{x}{2} - \frac{x}{3} \geq -\frac{5}{6}$$

$$\overset{3}{\cancel{6}}\left(\frac{x}{\underset{1}{\cancel{2}}}\right) - \overset{2}{\cancel{6}}\left(\frac{x}{\underset{1}{\cancel{3}}}\right) \geq \overset{1}{\cancel{6}}\left(-\frac{5}{\underset{1}{\cancel{6}}}\right) \quad \text{(Multiply every term by 6.)}$$

$$3x - 2x \geq -5$$

$$x \geq -5$$

Graph:



b.

$$\frac{1}{4} < \frac{x}{2} - \frac{x}{3}$$

$$\overset{3}{\cancel{12}}\left(\frac{1}{\cancel{4}}\right) < \overset{6}{\cancel{(12)}}\frac{x}{\cancel{2}} - \overset{4}{\cancel{(12)}}\frac{x}{\cancel{3}} \quad \text{(Multiply every term by 12.)}$$

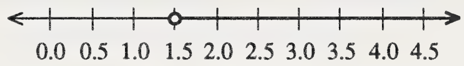
$$3 < 6x - 4x$$

$$3 < 2x$$

$$\frac{3}{2} < x \text{ or } x > \frac{3}{2} \quad \left(\frac{3}{2} = 1.5\right)$$

(Reverse the inequality sign when the variable is moved to the other side.)

Graph:



4. a.

$$\frac{(3-4x)}{2} \leq \frac{2(x-3)}{5} \quad \text{(Multiply both sides by 10.)}$$

$$5(3-4x) \leq 4(x-3)$$

$$15 - 20x \leq 4x - 12$$

$$15 - 20x - 4x \leq 4x - 12 - 4x$$

$$15 - 24x \leq -12$$

$$15 - 24x - 15 \leq -12 - 15$$

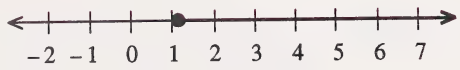
$$-24x \leq -27$$

$$\frac{-24x}{-24} \geq \frac{-27}{-24} \quad \text{(Divide both sides by } -24 \text{ and reverse the inequality sign.)}$$

$$x \geq \frac{27}{24}$$

$$x \geq \frac{9}{8} \text{ or } x \geq 1\frac{1}{8}$$

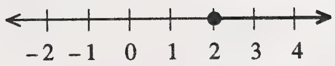
Graph:



b.

$$(x+2)(3x-1) \le 3x(x+2)-4 \quad \text{(Multiply the factors.)}$$
$$3x^2 - x + 6x - 2 \le 3x^2 + 6x - 4$$
$$3x^2 + 5x - 2 \le 3x^2 + 6x - 4$$
$$3x^2 - 3x^2 + 5x - 2 \le 3x^2 - 3x^2 + 6x - 4$$
$$5x - 2 \le 6x - 4$$
$$5x - 6x - 2 + 2 \le 6x - 6x - 4 + 2$$
$$-x \le -2$$
$$\frac{-x}{-1} \ge \frac{-2}{-1} \quad \text{(Divide each side by } -1 \text{ and reverse the inequality.)}$$
$$x \ge 2$$

Graph:



Activity 2

Solve word problems with linear inequalities.

1. Let x be the average mass of each passenger.

$$5x + 150 > 600$$
$$5x > 450 \quad \text{(Subtract 150 from both sides.)}$$
$$x > 90 \quad \text{(Divide both sides by 5.)}$$

The average mass of each passenger exceeds 90 kg.

2. Let x be the smaller number.
Then $x + 4$ is the larger number.

$$2x + (x+4) \le 25$$
$$3x + 4 \le 25$$
$$3x \le 21$$
$$x \le 7$$

The smaller number is less than or equal to 7.

3. Let x be a certain number.

$$3x + 5 < 14$$
$$3x + 5 - 5 < 14 - 5$$
$$3x < 9$$
$$x < 3$$

The solution is any number less than 3.

4. Let x be the water added.
Then $2+x$ is the volume of new solution.

$$\frac{30}{100}\times 2<\frac{10}{100}(2+x) \qquad \text{(Multiply each term by 100.)}$$
$$30\times 2<10(2+x)$$
$$60<20+10x$$
$$60-20<20+10x-20$$
$$40<10x$$
$$10x>40$$
$$x>4$$

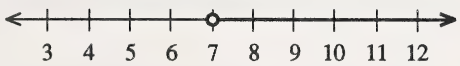
More than 4 L of water must be added.

Extra Help

1. The first step is to add 2 to both sides.

$$x-2+(\mathbf{2})>5+(\mathbf{2})$$
$$x>(\mathbf{7})$$

Graph:



Use an open dot because 7 is not included.

2. The first step is to subtract 1 from both sides.

$$x+1-(\mathbf{1})\le 3-(\mathbf{1})$$
$$x\le(\mathbf{2})$$

Graph:

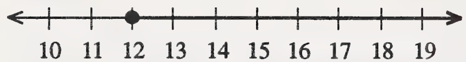
Use a solid dot because the number 2 is included.

3. The first step is to multiply both sides by 3.

$$\frac{x}{3}(\mathbf{3})\ge 4(\mathbf{3})$$
$$x\ge(\mathbf{12})$$

No, if you multiply both sides by a positive number, you do not have to reverse the inequality sign.

Graph:

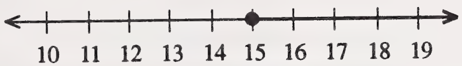


4. The first step is to multiply both sides by (-5).

$$\frac{x}{-5}(-5) \geq (-3)(-5)$$
$$x \geq (15)$$

Yes, if you multiply both sides by a negative number, you have to reverse the inequality sign.

Graph:

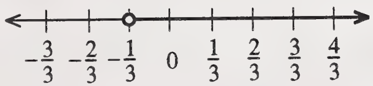


5. The first step is to divide both sides by 3.

$$\frac{3x}{(3)} > \frac{-1}{(3)}$$
$$x > \left(-\frac{1}{3}\right)$$

No, if you divide both sides by a positive number, you do not have to reverse the inequality sign.

Graph:

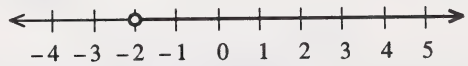


6. The first step is to divide both sides by (-3).

$$(-3x) < 6$$
$$\left(\frac{-3x}{-3}\right) > \left(\frac{6}{-3}\right)$$
$$x > (-2)$$

Yes, if you divide both sides by a negative number, you have to reverse the inequality sign.

Graph:

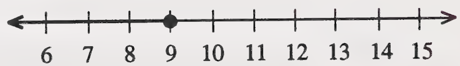


7. a. $x - 7 \leq 2$

$x - 7 + 7 \leq 2 + 7$ (Add 7 to both sides.)

$x \leq 9$

Graph:

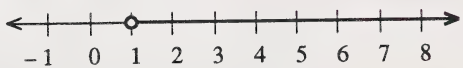


b. $x - 3 > -2$

$x - 3 + 3 > -2 + 3$ (Add 3 to both sides.)

$x > 1$

Graph:

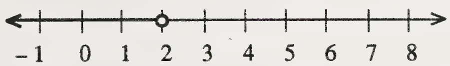


8. a. $x+1<3$

$x+1-1<3-1$ (Subtract 1 from both sides.)

$x<2$

Graph:

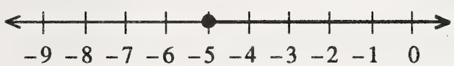


b. $x+2\ge -3$

$x+2-2\ge -3-2$ (Subtract 2 from both sides.)

$x\ge -5$

Graph:

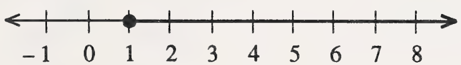


9. a. $\frac{x}{5}\ge\frac{1}{5}$

$\frac{x}{\underset{1}{\cancel{5}}}(\overset{1}{\cancel{5}})\ge\frac{1}{\underset{1}{\cancel{5}}}(\overset{1}{\cancel{5}})$ (Multipy both sides by 5.)

$x\ge 1$

Graph:



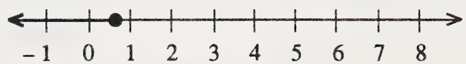
b. $\frac{x}{2}\le\frac{1}{3}$

$\overset{3}{\cancel{6}}\left(\frac{x}{\underset{1}{\cancel{2}}}\right)\le\overset{2}{\cancel{6}}\left(\frac{1}{\underset{1}{\cancel{3}}}\right)$ (Multiply both sides by 6.)

$3x\le 2$

$x\le\frac{2}{3}$ (Divide both sides by 3.)

Graph:



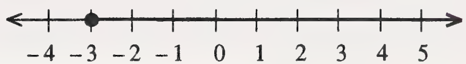
10. a. $\frac{-x}{18}\le\frac{1}{6}$

$\frac{-x}{\underset{1}{\cancel{18}}}(\overset{1}{\cancel{18}})\le\frac{1}{\underset{1}{\cancel{6}}}(\overset{3}{\cancel{18}})$ (Multiply both sides by 18.)

$-x\le 3$ (Multiply or divide both sides by -1

$x\ge -3$ and reverse the inequality sign.)

Graph:

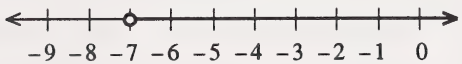


b. $\frac{x}{21} > \frac{-1}{3}$

$\cancel{(21)}^{1}\frac{x}{\cancel{21}_{1}} > \frac{-1}{\cancel{3}_{1}}\cancel{(21)}^{7}$

$x > -7$

Graph:



11. a. $3x+1>7$

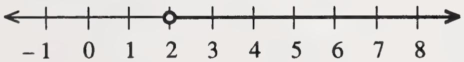
$3x+1-1>7-1$ (Subtract 1 from both sides.)

$3x>6$

$\frac{3x}{3} > \frac{6}{3}$ (Divide both sides by 3.)

$x>2$

Graph:



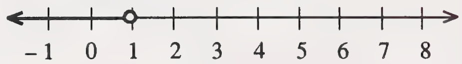
b. $5x+3<7$

$5x+3-3<7-3$ (Subtract 3 from both sides.)

$5x<4$

$x<\frac{4}{5}$ (Divide both sides by 5.)

Graph:



12. a. $-2x-1<-11$

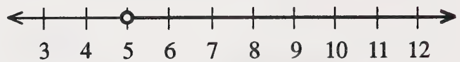
$-2x-1+1<-11+1$ (Add 1 to both sides.)

$-2x<-10$

$\frac{-2x}{-2} > \frac{-10}{-2}$ (Divide both sides by -2 and reverse the inequality sign.)

$x>5$

Graph:



b. $-3x+2>-10$

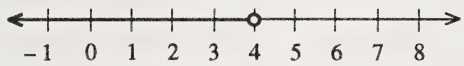
$-3x+2-2>-10-2$ (Subtract 2 from both sides.)

$-3x>-12$

$\frac{-3x}{-3}<\frac{-12}{-3}$ (Divide both sides by -3 and reverse the inequality sign.)

$x<4$

Graph:



13. Let x be the smaller even integer.
Then $x+2$ is the larger even integer.

$$x+(x+2)>38$$
$$2x+2>38$$
$$2x+2-2>38-2$$
$$2x>36$$
$$\frac{2x}{2}>\frac{36}{2}$$
$$x>18$$

Therefore, the smaller integer must be an even number greater than 18.

Extensions

It is given that $a>b>0$ and $c>d>0$.
The expression to prove is $ac>bd$.

Proof: If $a>b$, then $ac>bc$.
It is given that $a>b$. To get $ac>bc$, you multiply both sides by a positive number c.

If $c>d$, then $bc>bd$.
It is given that $c>d$. To get $bc>bd$, multiply both sides by b.

Since $bc=bc$, it follows that $ac>bc>bd$.
(If $a>b$ and $b>c$, then $a>c$.)
Thus, $ac>bd$.

Exploring Topic 3

Activity 1

Solve and verify simple quadratic equations which are easily reducible to $x^2=a$.

1. a. $x^2=49$

$$\sqrt{x^2}=\pm\sqrt{49}$$
$$x=\pm 7$$

Verify:

When $x = 7$:

LS	RS
7^2	49
49	49

LS = RS

When $x = -7$:

LS	RS
$(-7)^2$	49
49	49

LS = RS

b. $x^2 = 121$

$\sqrt{x^2} = \pm\sqrt{121}$

$x = \pm 11$

Verify:

When $x = 11$:

LS	RS
11^2	121
121	121

LS = RS

When $x = -11$:

LS	RS
$(-11)^2$	121
121	121

LS = RS

2. a. $5x^2 = 45$

$\frac{5x^2}{5} = \frac{45}{5}$

$x^2 = 9$

$\sqrt{x^2} = \pm\sqrt{9}$

$x = \pm 3$

Verify:

When $x = 3$:

LS	RS
$5(3)^2$	45
$5(9)$	45
45	45

LS = RS

When $x = -3$:

LS	RS
$5(-3)^2$	45
$5(9)$	45
45	45

LS = RS

b. $7a^2 = 252$

$\frac{7a^2}{7} = \frac{252}{7}$

$a^2 = 36$

$\sqrt{a^2} = \pm\sqrt{36}$

$a = \pm 6$

Verify:

When $a = 6$:

LS	RS
$7(6)^2$	252
$7(36)$	252
252	252

LS = RS

When $a = -6$:

LS	RS
$7(-6)^2$	252
$7(36)$	252
252	252

LS = RS

3. a. $7x^2-3=60$

$$7x^2-3+3=60+3$$
$$7x^2=63$$
$$x^2=9$$
$$\sqrt{x^2}=\pm\sqrt{9}$$
$$x=\pm 3$$

Verify:

When $x=3$:

LS	RS
$7(3)^2-3$	60
$7(9)-3$	60
$63-3$	60
60	60
LS =	RS

When $x=-3$:

LS	RS
$7(-3)^2-3$	60
$7(9)-3$	60
$63-3$	60
60	60
LS =	RS

b. $9x^2-2=47$

$$9x^2-2+2=47+2$$
$$9x^2=49$$
$$x^2=\frac{49}{9}$$
$$\sqrt{x^2}=\pm\sqrt{\frac{49}{9}}$$
$$x=\pm\frac{7}{3}$$

Verify:

When $x=\frac{7}{3}$:

LS	RS
$9\left(\frac{7}{3}\right)^2-2$	47
$9\left(\frac{49}{9}\right)-2$	47
$49-2$	47
47	47
LS =	RS

When $x=\frac{-7}{3}$:

LS	RS
$9\left(\frac{-7}{3}\right)^2-2$	47
$9\left(\frac{49}{9}\right)-2$	47
$49-2$	47
47	47
LS =	RS

4. a. $(3x-1)^2=196$

$$\sqrt{(3x-1)^2}=\pm\sqrt{196}$$
$$3x-1=\pm 14$$

Case 1: $3x-1=14$

$$3x=15$$
$$x=5$$

Verify:

LS	RS
$[3(5)-1]^2$	196
$(15-1)^2$	196
14^2	196
196	196

LS = RS

Case 2: $3x-1=-14$

$$3x=-13$$

$$x=-\frac{13}{3}$$

Verify:

LS	RS
$\left[3\left(-\frac{13}{3}\right)-1\right]^2$	196
$(-13-1)^2$	196
$(-14)^2$	196
196	196

LS = RS

b. $(5x+2)^2=1369$

$$\sqrt{(5x+2)^2}=\pm\sqrt{1369}$$

$$5x+2=\pm 37$$

Case 1: $5x+2=37$

$$5x=35$$

$$x=7$$

Verify:

LS	RS
$[5(7)+2]^2$	1369
$(35+2)^2$	1369
37^2	1369
1369	1369

LS = RS

Case 2: $5x+2=-37$

$$5x=-39$$

$$x=-\frac{39}{5}$$

Verify:

LS	RS
$\left[5\left(-\frac{39}{5}\right)+2\right]^2$	1369
$(-39+2)^2$	1369
$(-37)^2$	1369
1369	1369

LS = RS

5. $(x+3)^2 = 121$

$$\sqrt{(x+3)^2} = \pm\sqrt{121}$$
$$x+3 = \pm 11$$

Case 1: $x+3=11$

$x=8$

Case 2: $x+3=-11$

$x=-14$

Discard Case 2 since x cannot be negative.

$$\begin{aligned}\text{Area of the shaded rectangle} &= x(x+3)\\ &= 8(8+3)\\ &= 8\times 11\\ &= 88\end{aligned}$$

The area of the shaded rectangle is 88 cm^2.

Activity 2

Solve and verify simple radical equations which are easily reducible to $\sqrt{x}=b$.

1. a.

$$\begin{aligned}3\sqrt{y} &= 147\\ \frac{3\sqrt{y}}{3} &= \frac{147}{3}\\ \sqrt{y} &= 49\\ \left(\sqrt{y}\right)^2 &= (49)^2\\ y &= 2401\end{aligned}$$

Verify:

LS	RS
$3\left(\sqrt{2401}\right)$	147
$3(49)$	147
147	147

LS = RS

The solution is $y=2401$.

b.

$$\begin{aligned}5\sqrt{x} &= 320\\ \frac{5\sqrt{x}}{5} &= \frac{320}{5}\\ \sqrt{x} &= 64\\ \left(\sqrt{x}\right)^2 &= (64)^2\\ x &= 4096\end{aligned}$$

Verify:

LS	RS
$5\left(\sqrt{4096}\right)$	320
$5(64)$	320
320	320

LS = RS

The solution is $x=4096$.

2. a.

$$\begin{aligned} 2\sqrt{x}-3&=19 \\ 2\sqrt{x}-3+3&=19+3 \\ 2\sqrt{x}&=22 \\ \sqrt{x}&=11 \\ \left(\sqrt{x}\right)^2&=11^2 \\ x&=121 \end{aligned}$$

Verify:

LS	RS
$2\left(\sqrt{121}\right)-3$	19
$2(11)-3$	19
$22-3$	19
19	19

LS = RS

The solution is $x=121$.

b.

$$\begin{aligned} 7\sqrt{x}-5&=23 \\ 7\sqrt{x}-5+5&=23+5 \\ 7\sqrt{x}&=28 \\ \sqrt{x}&=4 \\ \left(\sqrt{x}\right)^2&=4^2 \\ x&=16 \end{aligned}$$

Verify:

LS	RS
$7\left(\sqrt{16}\right)-5$	23
$7(4)-5$	23
$28-5$	23
23	23

LS = RS

The solution is $x=16$.

3. a.

$$\begin{aligned} \sqrt{3x-2}&=5 \\ \left(\sqrt{3x-2}\right)^2&=5^2 \\ 3x-2&=25 \\ 3x&=27 \\ x&=9 \end{aligned}$$

Verify:

LS	RS
$\sqrt{3(9)-2}$	5
$\sqrt{27-2}$	5
$\sqrt{25}$	5
5	5

LS = RS

The solution is $x=9$.

b. $\sqrt{6x-3}=3$

$$\left(\sqrt{6x-3}\right)^2=3^2$$
$$6x-3=9$$
$$6x-3+3=9+3$$
$$6x=12$$
$$x=2$$

Verify:

LS	RS
$\sqrt{6(2)-3}$	3
$\sqrt{12-3}$	3
$\sqrt{9}$	3
3	3

LS = RS

The solution is $x=2$.

4. a. $3\sqrt{x}+5=2$

$$3\sqrt{x}+5-5=2-5$$
$$3\sqrt{x}=-3$$
$$\sqrt{x}=-1$$
$$\left(\sqrt{x}\right)^2=(-1)^2$$
$$x=1$$

Verify:

LS	RS
$3\left(\sqrt{1}\right)+5$	2
$3(1)+5$	2
$3+5$	2
8	2

LS $\neq$ RS

Therefore, $x=9$ is **not** a real solution.

b. $8\sqrt{x}+46=6$

$$8\sqrt{x}+46-46=6-46$$
$$8\sqrt{x}=-40$$
$$\sqrt{x}=-5$$
$$x=25$$

Verify:

LS	RS
$8\left(\sqrt{25}\right)+46$	6
$8(5)+46$	6
$40+46$	6
86	6

LS $\neq$ RS

Therefore, $x=25$ is **not** a real solution.

5.

$$4 = 2 \times 3.1416\sqrt{\frac{l}{9.8}}$$

$$4 = 6.2832\sqrt{\frac{l}{9.8}}$$

$$\frac{4}{6.2832} = \sqrt{\frac{l}{9.8}}$$ (Keep all the digits on your calculator in the intermediate steps.)

$$0.636\,618\,283 = \sqrt{\frac{l}{9.8}}$$

$$(0.636\,618\,283)^2 = \left(\sqrt{\frac{l}{9.8}}\right)^2$$

$$\frac{l}{9.8} = 0.405\,282\,839$$

$$\frac{l}{9.8} \times 9.8 = 0.405\,282\,839 \times 9.8$$

$$l = 3.971\,771\,823$$

$$\doteq 3.97$$

The length of the pendulum is 3.97 m.

Extra Help

1. a. 56 b. 124
2. a. 5.3 b. 0.365
3. a. 3271.84 b. 3969
4. a. 0.0441 b. 0.2809

Extensions

1. Find $\sqrt{1681}$.

$$\begin{array}{r} 4 \times 20 = 80 \\ +\ 1 \\ \hline 81 \end{array}$$

	4	1	.
4	16	81	.
	16		
	0	81	
		81	
		0	

Therefore, $\sqrt{1681} = 41$.

2. Find $\sqrt{1049.76}$.

$$\begin{array}{r} 3 \times 20 = 60 \\ +\ 2 \\ \hline 62 \end{array}$$

$$\begin{array}{r} 32 \times 20 = 640 \\ +\ 4 \\ \hline 644 \end{array}$$

	3	2	.4
3	10	49	.76
	9		
	1	49	
	1	24	
		25	76
		25	76
			0

Therefore, $\sqrt{1049.76} = 32.4$.

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Mathematics 10

Student Module
Unit 3

L.R.D.C.
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